

Jump-Diffusion Option Valuation and Option-Implied Investor Preferences: A Stochastic Dominance Approach

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Abstract

We investigate the relationship between the discrete-time stochastic dominance option bounds and the continuous time arbitrage-based option pricing models when the underlying asset returns follow a jump-diffusion. Building upon the stochastic dominance approach, we derive multipored index option bounds, violations of which trigger investment strategies that increase the expected utility of any risk-averse trader by introducing a corresponding short (long) option in her portfolio. As trading becomes more frequent, we provide empirical evidence that the bounds converge into a tight limit interval that includes the Merton jump-diffusion price, and compares favorably to the observed bid-ask spreads in option markets. For CRRA investors we also examine the limits of the admissible values of the relative risk aversion coefficient compatible with the boundary risk-neutral distributions extracted from underlying index return data. We show that, unlike the option prices derived from an equilibrium jump diffusion model in both underlying and option markets, the SD bounds can better accommodate reasonable values of the ex-dividend expected excess return. Moreover, the SD-restricted range of admissible RRA values is consistent with the recent macro-finance studies of the equity premium puzzle that show the relative importance of rare disasters in the consumption distribution in explaining the observed equity premium.

(JEL G12, G13)

Keywords. stochastic dominance; option pricing; option bounds; incomplete markets; jump diffusion; risk aversion;

1 Introduction

In this paper we present a model of the derivation of the risk neutral or Q -distribution for an asset whose returns follow jump diffusion asset dynamics. Such derivations in previous studies have relied on general equilibrium considerations involving both the underlying and the option market data, in which the unknown parameters are estimated from both sets of observed market prices. A key input in these derivations is the risk aversion parameter of a representative investor, whose preferences are almost always assumed to be of the Constant Relative Risk Aversion (CRRA) type. Unlike these approaches our model requires only underlying market data and uses a weaker concept than equilibrium, that of stochastic dominance (SD); it also does not assume the existence of a representative investor. We also use our approach in order to assess the appropriateness of the various values of the risk aversion parameter that have been used in earlier studies in conjunction with the asset dynamics parameters extracted from econometric studies in the underlying market.

Equilibrium models are established either based on the production economy or on the exchange economy. In a production setting a representative investor chooses her optimal level of consumption in each period and invests the rest in the production for future consumption, where the production technology grows stochastically and the initial endowment is constant. The large literature on this model includes [Brock \[1982\]](#), [Cox et al. \[1985\]](#), [Cochrane \[1991\]](#), and [Cochrane \[1996\]](#). Studies that consider jumps in the production process are [Ahn and Thompson \[1988\]](#), [Bates \[1988\]](#), and [Bates \[1991\]](#). [Pan \[2002\]](#), [Liu et al. \[2003\]](#), and [Zhang et al. \[2012\]](#) also include jumps in a production economy but in a partial equilibrium setting as they only study the price of derivatives and disregard the price of assets. In addition, there are several equilibrium studies in an exchange economy based on consumption asset pricing, where aggregate endowment is stochastic such as, among others, [Lucas Jr \[1978\]](#), [Breedon \[1979\]](#), and more recently [Bates \[2008\]](#), and [Santa-Clara and Yan \[2010\]](#).

By contrast, the stochastic dominance literature is slimmer, even though it appeared more than 30 years ago. It was first introduced by [Perrakis and Ryan \[1984\]](#), [Levy \[1985\]](#), [Ritchken \[1985\]](#), and subsequently extended by [Perrakis \[1986\]](#) and [Ritchken and Kuo \[1988\]](#). More recently [Constantinides and Perrakis \[2002\]](#) and [Constantinides and Perrakis \[2007\]](#) extended it to incorporate proportional transaction costs, an extension that was tested empirically in [Constantinides et al. \[2009\]](#) and [Constantinides et al. \[2011\]](#). Jump diffusion valuation elements for a specific type of insurance derivatives were applied in the SD context in [Perrakis and Bolorforoosh \[2013\]](#), while [Oancea and Perrakis \[2014\]](#) (OP, 2014) established the formal equivalence of SD to the [Black and Scholes \[1973\]](#) model under simple diffusion asset dynamics for both index and equity options.

This paper presents the SD theory for index options in a general jump diffusion context and examines the equilibrium models' results within the more general framework of SD. We derive upper and lower bounds on option prices based on the parameters of the physical distribution of the underlying return process. We then compare these bounds to equilibrium

models' predicted option values and the associated risk neutral volatility and mean return of the underlying asset as functions of the relative risk aversion (RRA) parameter. We use jump diffusion asset dynamics parameters extracted from available econometric studies in the S&P 500 underlying index market. We rely on the fact that the SD bounds are independent of RRA but rely, on the other hand, on the ex-dividend mean return of the underlying asset; as we point out, for the most frequently used underlying, the S&P 500 index, that range is widely assumed to lie within known limits. Further, we observe that the derived bounds are relatively insensitive to the parameters of the jump component provided the total volatility is kept constant; this is important because there is a large variability in the estimates of these parameters depending on the time span of the data.¹

By contrast, the econometric literature has presented widely divergent values of the RRA coefficient. Even within the option pricing models and associated empirical research the RRA coefficient varies widely between studies and even within the same study.² As we show in this paper, several of these high-end RRA values yield economically meaningless results within any equilibrium model, since either the option price or the implied mean return are beyond any reasonable values. This is true even a fortiori for RRA estimates extracted out of the equity premium puzzle literature, which can be more than five times as large.³ By contrast, we show that the SD bounds extracted from several econometric estimations of jump diffusion parameters for the S&P 500 index are consistent with RRA estimates as high as the equity premium ones, the only option price-implied estimates to possess this property; in fact, the puzzle disappears in SD-implied RRA parameters and associated mean returns. Last, we show that the SD bounds' implied RRA is also consistent with the more recent stylized models in the equity premium studies that include rare events in a representative investor's consumption growth in an attempt to reconcile the estimates with observed quantities and solve the puzzle.⁴

The paper proceeds as follows. Section 2 presents the jump diffusion stochastic dominance bounds as the limits of the discrete time SD bounds following a modified version of the approach in [Oancea and Perrakis \[2014\]](#). Section 3 presents a summary of the dominant equilibrium approach and extracts the implied bounds on the RRA parameter from the SD option bounds. Section 4 applies these results in several empirically important cases and shows that the SD bounds can reconcile several of the apparently puzzling results derived by earlier studies in option markets or in the equity premium puzzle literature. Section 5 concludes. In the appendix we discuss the implications of combining jump processes with stochastic volatility (SV) diffusion, which can be handled with SD as long as the pricing of the systematic risk of SV is done independently.⁵

¹See, for instance, [Andersen et al. \[2002, Tables 3 and 6\]](#) and [Tauchen and Zhou \[2011, Table 4\]](#).

²See, for instance, [Rosenberg and Engle \[2002\]](#) and [Bliss and Panigirtzoglou \[2004\]](#), who find coefficients ranging from 2 to 12 and 1.97 to 7.91 respectively.

³The reported RRA estimates are 41 for [Mehra and Prescott \[1985\]](#), 40 to 50 for [Cochrane and Hansen \[1992\]](#), and more than 35 for [Campbell and Cochrane \[1999\]](#).

⁴See, for instance, [Barro \[2009\]](#), [Wachter \[2013\]](#), [Backus et al. \[2011\]](#) and [Martin \[2013\]](#).

⁵In this respect the SD approach is no different from the alternative equilibrium approach. See, for

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The SD approach derives upper and lower bounds on the option prices in a multiperiod discrete time context and then finds the limits of these bounds as the time partition tends to zero. The derivation of the bounds was done in earlier studies, most recently in [Oancea and Perrakis \[2014\]](#) and will not be repeated here. We summarize the results and assumptions of the SD model before applying them to jump diffusion.

In a discrete time model trading occurs at a finite number of trading dates $t = 0, 1, \dots, T$ of length Δt . We consider an index as the underlying asset with current price S_t and return $(S_{t+\Delta t} - S_t)/S_t \equiv z_{t+\Delta t}$ in each time interval. We also consider a riskless asset with return equal to R in each time period with r as a continuous time counterpart of return, where $(1 + R) = e^{r\Delta t} = 1 + r\Delta t + o(\Delta t)$. The SD bounds are derived under the following set of assumptions.

1. There exists at least one utility-maximizing risk averse investor (the trader) in the economy who holds only the index and the riskless asset;⁶
2. This particular investor is marginal in the option market;
3. The riskless rate is non-random.⁷

These market equilibrium assumptions are quite general, insofar as they do not require that all agents have the properties that we assign to traders, thus allowing a market with heterogeneous agents and the existence of other investors with different portfolio holdings than the trader.

Let $P(z_{j,t+\Delta t})$ denotes the physical return distribution, assumed continuous without loss of generality. By assumption, $E[z_{t+\Delta t} | S_t] > R$.⁸ Similarly, let $z_{\min,t+\Delta t}$ denotes the lowest possible return, which is initially assumed to be strictly greater than -1 . In our equilibrium, we also define the upper (lower) bounds, $\bar{C}_t(S_t)$ ($\underline{C}_t(S_t)$), on the admissible call option

instance, [Liu et al. \[2005, footnote 9\]](#).

⁶This assumption implies that the pricing kernel in any multiperiod equilibrium model is a monotone decreasing function of the return.

⁷As discussed in [Oancea and Perrakis \[2014\]](#), although the constant riskless rate may not be justified in practice, its effect on option values is generally recognized as minor in short- and medium-lived options. It has been adopted without any exception in all equilibrium based jump-diffusion option valuation models that have appeared in the literature. See the comments in [Bates \[1991, note 30\]](#) and [Amin and Ng \[1993, P.891\]](#). [Bakshi et al. \[1997\]](#) found that stochastic interest rates do not improve the goodness of fit in a model featuring stochastic volatility and jumps.

⁸When the underlying asset is the index, as Section 2, this assumption is always true. However, when we have stocks with negative beta and non-decreasing pricing kernel, the equivalent assumption would be $\hat{z}_{n,t+\Delta t} < R$.

prices as the reservation write (purchase) prices of the option under market equilibrium that excludes the presence of stochastically dominant strategies. Violations of the bounds trigger investment strategies that increase the expected utility of any trader by introducing a corresponding short (long) option in her portfolio.

To derive the bounds $\bar{C}_t(S_t)$ and $\underline{C}_t(S_t)$ we recursively apply the Lemma 1 and Proposition 1 in [Oancea and Perrakis \[2014\]](#). Note that the derivation of the bounds depends on the convexity of the call option prices and payoff, a property which clearly holds for the jump-diffusion dynamics as well.⁹

Lemma 1. *If the option price $C_t(S_t)$ is convex for any t then it lies within the following bounds:*

$$\frac{1}{1+R} E^{L_t} [C_{t+\Delta t}(S_t(1+z_{t+\Delta t}))] \leq C_t(S_t) \leq \frac{1}{1+R} E^{U_t} [C_{t+\Delta t}(S_t(1+z_{t+\Delta t}))], \quad (2.1)$$

where E^{U_t} and E^{L_t} denote respectively expectations taken with respect to the distributions

$$\begin{aligned} U(z_{t+\Delta t}) &= \begin{cases} P(z_{t+\Delta t} | S_t) & \text{with probability } \frac{R - z_{\min, t+\Delta t}}{E(z_{t+\Delta t}) - z_{\min, t+\Delta t}} \\ 1_{z_{\min, t+\Delta t}} & \text{with probability } \frac{E(z_{t+\Delta t}) - R}{E(z_{t+\Delta t}) - z_{\min, t+\Delta t}} \end{cases} \equiv Q \\ L(z_{t+\Delta t}) &= P(z_{t+\Delta t} | S_t, z_{t+\Delta t} \leq z_t^*) \end{aligned} \quad (2.2)$$

Proof. See Lemma 1 in [Oancea and Perrakis \[2014\]](#). □

Remark 1. *Note that U_t and L_t are risk neutral as $E^{U_t}(1+z_{t+\Delta t}) = E^{L_t}(1+z_{t+\Delta t}) = R$, that is the distributions U_t and L_t are the incomplete market counterparts of the risk neutral probabilities of the binomial model.* □

Remark 2. *The distributions U_t and L_t depend on the entire distribution of the underlying asset and not only on its volatility parameter, as in the binomial and the BSM models.* □

Remark 3. *Note also that the upper bound pricing kernel, which is related to U_t , spikes at $z_{\min, t+\Delta t}$ and is constant thereafter while the lower bound pricing kernel, which is related to L_t , is zero for $z_{t+\Delta t} > z_t^*$ and constant positive elsewhere. We will discuss more about these two pricing kernels in Section 3.* □

⁹The convexity of the option with respect to the underlying stock price holds in all cases in which the return distribution has *i.i.d.* time increments, in all univariate state-dependent diffusion processes, and in bivariate (stochastic volatility) diffusions under most assumed conditions; see [Merton \[1973\]](#) and [Bergman et al. \[1996\]](#).

Proposition 1. *Under the monotonicity of the pricing kernel assumption and for a discrete distribution of the stock return z_t , all admissible option prices lie between the upper and lower bounds $\overline{C}_t(S_t)$ and $\underline{C}_t(S_t)$, evaluated by the following recursive expressions*

$$\begin{aligned}\overline{C}_T(S_T) &= C_T(S_T) = (S_T - K)^+ \\ \overline{C}_t(S_t) &= \frac{1}{1+R} E^{U_t} [\overline{C}_{t+\Delta t}(S_t(1+z_{t+\Delta t})) | S_t] \\ \underline{C}_t(S_t) &= \frac{1}{1+R} E^{L_t} [\underline{C}_{t+\Delta t}(S_t(1+z_{t+\Delta t})) | S_t]\end{aligned}\tag{2.3}$$

where E^{U_t} and E^{L_t} denote expectations taken with respect to the distributions given in (2.2).

Proof. See Proposition 1 in [Oancea and Perrakis \[2014\]](#). □

Remark 4. *Note that in the special case where a stock can become worthless within a single elementary time period $(t, t + \Delta t)$ we have $z_{\min, t+\Delta t} = -1$, irrespective of the underlying index dynamics. In such a case the upper bound distribution is no longer risk neutral and can be extracted by (2.4) where the expectation is taken with respect to the actual distribution $P(z_{t+\Delta t} | S_t)$ rather than the upper bound distribution in (2.2). □*

$$\begin{aligned}\overline{C}_T(S_T) &= (S_T - K)^+ \\ \overline{C}_t(S_t) &= \frac{E^P [\overline{C}_{t+\Delta t}(S_t(1+z_{t+\Delta t})) | S_t]}{E[1+z_{t+\Delta t} | S_t]}\end{aligned}\tag{2.4}$$

This important special case yields a loose upper bound on the call option prices but also a convenient closed form solution when the underlying return follows jump-diffusion dynamics.

We model the returns as a sum of two components, one of which will tend to a diffusion with a probability of $1 - \lambda_t \Delta t$, and the other to a jump process. Therefore, the return dynamic has the following form.¹⁰

$$z_{t+\Delta t} = \begin{cases} [\mu(S_t, t) - \lambda_t k] \Delta t + \sigma(S_t, t) \epsilon \sqrt{\Delta t} & \text{with probability } (1 - \lambda_t \Delta t) \\ [\mu(S_t, t) - \lambda_t k] \Delta t + \sigma(S_t, t) \epsilon \sqrt{\Delta t} + (j_t - 1) & \text{with probability } (\lambda_t \Delta t) \end{cases}\tag{2.5}$$

In this expression ϵ has a bounded distribution of mean zero and variance one, $\epsilon \sim D(0, 1)$ and $0 < \epsilon_{\min} \leq \epsilon \leq \epsilon_{\max}$, but otherwise unrestricted. With probability $\lambda_t \Delta t$ there is a jump

¹⁰For simplicity dividends are ignored throughout this paper. All results can be easily extended to the case where the stock has a known and constant dividend yield, as in index options. In the latter case the instantaneous mean in (2.6) and (2.8) is net of the dividend yield.

with amplitude equal to j_t . This amplitude is a random variable with distribution $j_t \sim D_{j_t}(\mu_{j_t}, \sigma_{j_t})$. Although our results may be extended to allow for dependence of both jump intensity and jump amplitude distributions on S_t , we shall adopt the common assumption in the literature that the jump process is state- and time-independent, with $\lambda_t = \lambda$, $j_t = j$. Similarly, it is commonly assumed that jump amplitude is log-normally distributed, implying that $J = \ln(j) \sim N(\mu_j - \frac{1}{2}\sigma_j^2, \sigma_j^2)$ with $\mu_j = \ln(E[j])$ and $k = e^{\mu_j} - 1$. In our case we adopt more general assumptions, with the distribution D_j restricted to a non-negative support, so that the variable j takes values with $0 \leq j_{\min}$ but otherwise unrestricted. With this specification the return becomes, if we set $\mu(S_t, t) \equiv \mu_t$, $\sigma(S_t, t) \equiv \sigma_t$

$$z_{t+\Delta t} = (\mu_t - \lambda k)\Delta t + \sigma_t \epsilon \sqrt{\Delta t} + (j - 1)\Delta N, \quad (2.6)$$

where N is a Poisson counting process with intensity λ . Following the proposed specification, the return distribution in (2.5) is replaced with the one introduced in (2.7), which will also be used in the proofs.

$$z_{t+\Delta t} = \begin{cases} [\mu_t - \lambda k] \Delta t + \sigma_t \epsilon \sqrt{\Delta t} & \text{with probability } (1 - \lambda \Delta t) \\ [\mu_t - \lambda k] \Delta t + \sigma_t \epsilon \sqrt{\Delta t} + (j - 1) & \text{with probability } (\lambda \Delta t) \end{cases} \quad (2.7)$$

In the remainder of this section we first present conditions that establish the convergence of the return process described in (2.6) and (2.7) to a mixed jump-diffusion process. We then extract the two option bound distributions from (2.1) and (2.2) and find their convergence to continuous time expressions following the approach in OP (2014) for convergence of (2.6) to a diffusion process in the absence of jumps. That approach defines a sequence of stock prices and associated probability measures and proves that the proposed sequence converges¹¹ in distribution to a diffusion and its probability converges weakly to the respected probability measure. Therefore, mean and variance of the discrete process converge weakly to the equivalent parameters of the diffusion process. In the case of jump diffusion we may prove the following lemma. Then we close this section by numerical analysis regarding the proposed upper and lower bounds on the option prices.

Lemma 2. *The discrete process described by (2.6) converges weakly to the jump-diffusion process (2.8) as the time interval approaches to zero.*

$$dS_t/S_t = (\mu_t - \lambda k) dt + \sigma_t dW + (j - 1) dN \quad (2.8)$$

Proof. See Appendix A. □

¹¹More details on the weak convergence and its properties for Markov processes can be found at Ethier and Kurtz [2009], or Stroock and Varadhan [2007].

For the discrete time process (2.6), which tends to a jump-diffusion (2.8), a unique option price can be derived by arbitrage methods alone only if we have zero volatility and the jump amplitude takes exactly one value when a jump occurs. In such a case the process (2.6) is binomial and it can be readily verified that the upper bound distributions, U_t , and the lower bound distribution, L_t , coincide and the stochastic dominance approach yields the same unique option price as the binomial jump process in Cox et al. [1979]. Otherwise, we must examine the two bounds separately.

For the option upper bound we apply the transformation (2.2) to the discretization (2.6), assuming first that $j_{\min} > 0$. For such a process we note that as Δt decreases, there exists h , such that for any $\Delta t \leq h$, the minimum outcome of the jump component is less than the minimum outcome of the diffusion component, $(j_{\min} - 1) < (\mu_t \Delta t + \sigma_t \epsilon_{\min} \sqrt{\Delta t})$. Consequently, for any $\Delta t \leq h$, the minimum outcome of the returns distribution is $(j_{\min} - 1)$, which is the value that we substitute for the minimum return, $z_{\min, t+\Delta t}$, in the transformation (2.2). With such a substitution we have now the following result for the jump diffusion upper bound on the call option price.

Proposition 2. *When the underlying asset follows a jump-diffusion process described by (2.8) the upper option bound is the expected payoff discounted by the riskless rate of an option on an asset whose dynamics are described by the jump-diffusion process*

$$dS_t/S_t = (r - (\lambda + \lambda_{U_t}) k^U) dt + \sigma_t dW_t + (j_t^U - 1) dN_t^Q, \quad (2.9)$$

where the upper bound risk-neutral jump intensity is $\lambda^U = \lambda + \lambda_{U_t}$ and

$$\lambda_{U_t} = -\frac{\mu_t - r}{j_{\min} - 1}, \quad (2.10)$$

and j_t^U is a mixture of jumps with intensity $\lambda + \lambda_{U_t}$ and distribution and mean

$$j_t^U = \begin{cases} j & \text{with probability } \frac{\lambda}{\lambda + \lambda_{U_t}} \\ j_{\min} & \text{with probability } \frac{\lambda_{U_t}}{\lambda + \lambda_{U_t}} \end{cases} \quad (2.11)$$

$$E[j_t^U - 1] = k^U = \left(\frac{\lambda}{\lambda + \lambda_{U_t}}\right)k + \left(\frac{\lambda_{U_t}}{\lambda + \lambda_{U_t}}\right)(j_{\min} - 1)$$

Proof. See Appendix B. □

By definition of the convergence of the discrete time process, *Proposition 2* states that the call upper bound is the discounted expectation of the call payoff under the risk neutral jump-diffusion process given by (2.9). We may, therefore, use the results derived by

Merton [1976] for options on assets following jump-diffusion processes with the jump risk fully diversifiable.¹² Applying Merton's approach to the jump-diffusion process given by (2.9), we find that the upper bound on call option prices for the jump-diffusion process (2.8) must satisfy the following partial differential equation (PDE), with terminal condition $C(S_T, T) = \max \{S_T - K, 0\}$:

$$\frac{1}{2}\sigma_t^2 S^2 \frac{\partial^2 \bar{C}}{\partial S^2} + [r - (\lambda + \lambda_{U_t})k^U] S \frac{\partial \bar{C}}{\partial S} - \frac{\partial \bar{C}}{\partial T} + (\lambda + \lambda_{U_t})E^U [\bar{C}(Sj_t^U) - \bar{C}(S)] = r\bar{C} \quad (2.12)$$

An important special case of the upper bound is when the lower limit of the jump amplitude is equal to 0, in which case $j_{\min} = 0$ and the return distribution has an absorbing state in which the stock becomes worthless and so the lowest possible return would be $z_{1t+\Delta t} = z_{\min, t+\Delta t} = -1$; this is the case described in the Remark 4 and equation (2.4), in which as we saw the option upper bound is the expected payoff with the actual distribution, discounted by the expected return on the stock. Hence, this is identical to the Merton [1976, Equation 14] case with r replaced by μ , yielding

$$\frac{1}{2}\sigma_t^2 S^2 \frac{\partial^2 \bar{C}}{\partial S^2} + [\mu_t - \lambda k] S \frac{\partial \bar{C}}{\partial S} - \frac{\partial \bar{C}}{\partial T} + \lambda E^U [\bar{C}(Sj_t^U) - \bar{C}(S)] = \mu_t \bar{C}. \quad (2.13)$$

If (2.13) holds and as in Bates [1991] we assume, in addition, that the diffusion parameters are constant and the jumps amplitude has a lognormal distribution with $\ln(j) \sim N(\mu_j - \frac{1}{2}\sigma_j^2, \sigma_j^2)$ where $k = E[j - 1] = e^{\mu_j} - 1$, then the distribution of the asset prices given that n jumps occurred is conditionally normal, with the following mean and variance.

$$\begin{aligned} \mu_n &= \mu - \lambda k + \frac{n}{T}\mu_j \\ \sigma_n^2 &= \sigma^2 + \frac{n}{T}\sigma_j^2 \end{aligned} \quad (2.14)$$

Hence, if n jumps occurred, the option price would be a Black-Scholes expression with μ_n replacing the riskless rate r , or $BS(S, X, T, \mu_n, \sigma_n)$. Integrating (2.13) would then yield the following upper bound, which can be obtained directly from Merton [1976] by replacing r by μ .

¹²Note that we do not assume here that the jump risk is diversifiable.

$$\begin{aligned}
\bar{C}(S, X, T, \mu_n, \sigma_n) &= \sum_{n=0}^{\infty} \frac{e^{\lambda'T} (\lambda'T)^n}{n!} [SN(d_{1n}) - X e^{-\mu_n T} N(d_{2n})] \\
d_{1n} &= \frac{\ln(S/X) + (\mu_n + 0.5\sigma_n^2)T}{\sqrt{\sigma_n^2 T}}, \quad d_{2n} = d_{1n} - \sqrt{\sigma_n^2 T} \\
\ln(j) &\sim N[\mu_j - 0.5\sigma_j^2, \sigma_j^2], \quad j \sim \text{lognormal}[e^{\mu_j}, e^{2\mu_j}(e^{\sigma_j^2} - 1)] \\
k &= E[j - 1], \quad k = e^{\mu_j} - 1, \quad \ln(1 + k) = \mu_j, \quad \lambda' = \lambda(1 + k)
\end{aligned} \tag{2.15}$$

When the jump distribution is not normal, the conditional asset distribution given n jumps is the convolution of a normal and n jumps distribution. The upper bound cannot be obtained in closed form, but it is possible to obtain the characteristic function of the bounds distributions. We will extract the bound's characteristic functions, its pricing kernels, and the respected properties in the next section. Similar approaches can be applied to the integration of equation (2.13), which holds whenever $-1 < (j_{\min} - 1) < 0$. Closed form solutions can also be found whenever the amplitude of the jumps is fixed as, for instance, when there is only an up or a down jump of a fixed size. A PDE similar to (2.13) also holds if the process has only "up" jumps, in which case $(j_{\min} - 1) = 0$ and the lowest return z_{\min} in (2.2) comes from the diffusion component. In such a case the key probability Q of (2.2) is the same as in the case of diffusion, discussed in the proof of Proposition 2 of Oancea and Perrakis [2014]. In this situation, equation (2.12) still holds with $\lambda_{U_t} = 0$, implying that the option upper bound is the Merton [1976] bound, with the jump risk fully diversifiable.

The option lower bound for the jump-diffusion process given by (2.8) and its discretization (2.6) is found by a similar procedure. We apply $L(z_{t+\Delta t})$ from (2.2) to the process (2.6) and we prove in the appendix the following result.

Proposition 3. *When the underlying asset follows a jump-diffusion process described by (2.8), the lower option bound is the expected payoff discounted by the riskless rate of an option on an asset whose dynamics is described by the jump-diffusion process*

$$dS_t/S_t = [r - \lambda k^L] dt + \sigma_t dW + (j_t^L - 1) dN \tag{2.16}$$

where the lower bound's jump intensity remains the same, $\lambda k^L = \lambda$, and j_t^L is absolute jump size with the truncated distribution $j|j \leq \bar{j}_t$.

The mean of the relative jump size, k^L , and the value of truncation boundary, \bar{j}_t , can be obtained by solving the following equations.

$$\begin{aligned}
\mu_t - r &= \lambda k - \lambda k^L \\
k^L &= E(j - 1 | j \leq \bar{j}_t)
\end{aligned} \tag{2.17}$$

Proof. See Appendix C. □

Observe that (2.16) always has a solution since $\mu_t > r$ by assumption. The limiting distribution includes the whole diffusion component and a truncated jump component. Unlike simple diffusion, the truncation does not disappear as $\Delta t \rightarrow 0$. As with the upper bound, we can apply the Merton [1976] approach to derive the PDE satisfied by the option lower bound, which is given by

$$\frac{1}{2}\sigma_t^2 S^2 \frac{\partial^2 \underline{C}}{\partial S^2} + [r - \lambda k^L] S \frac{\partial \underline{C}}{\partial S} - \frac{\partial \underline{C}}{\partial T} + \lambda E^L [\underline{C}(S_{j_t^L}) - \underline{C}(S)] = r \underline{C} \quad (2.18)$$

with terminal condition $\underline{C}_T = C(S_T, T) = \max\{S_T - K, 0\}$. The solution of (2.18) can be obtained in closed form only when the jump amplitudes are fixed, since even when the jumps are normally distributed, the lower bound jump distribution is truncated.

Observe that the jump components in both $\overline{C}_t(S_t)$ and $\underline{C}_t(S_t)$ are now state-dependent if μ_t , the diffusion component of the instantaneous expected return on the stock, is state-dependent, even though the actual jump process is independent of the diffusion. In many empirical applications of jump-diffusion processes, which were on the S&P 500 index options, the unconditional estimates are considered unreliable. On the other hand there is consensus that the unconditional mean is in the 4–6% range;¹³ this is reflected in the numerical results. Observe also that for normally distributed jumps the only parameters that enter into the computation of the bounds are the mean of the process, the volatility of the diffusion and the parameters of the jump component. Hence, the information requirements are the same as in the more traditional approaches, with the important difference that the mean of the process replaces the risk aversion parameter. This difference favors the SD approach, as the consensus that exists for the values of the mean of the process does not extend to the risk aversion parameter, as we shall see in the next section.

We illustrate in Table 2.1 and Figure 2.1, the convergence of the bounds under a jump-diffusion process for an ATM option with $X = 100$, time to maturity $T = 0.25$ years, and the annual parameters: $r = 2\%$, $\mu = 4\%$, $\sigma = 20\%$, $\lambda = 0.6$, $\mu_j = -0.05$, $\sigma_j = 7\%$. In our numerical analysis, the diffusion process was approximated by a sequence of trinomial trees constructed according to the algorithm of Kamrad and Ritchken [1991]. The jump process was approximated by a sequence of multinomial trees with up to 1000 time periods, which is based on the algorithm of Amin [1993], where the jump amplitude distribution is lognormal. For each tree, the upper and lower bound risk-neutral probability distributions were computed by applying equation (2.2) respectively to the single-period distribution. The two option bounds were evaluated as discounted expectations of the option payoff under the two risk neutral distributions described in Propositions (2) and (3). In order to evaluate the bounded jump amplitudes discussed in the case where $j_{min} > 0$, the distribution was

¹³See Fama and French [2002], Constantinides [2002] and Dimson et al. [2006].

truncated to a worst-case jump return of -20% . The truncation limit is chosen to meet the observed jump amplitude in econometric studies of jump diffusion. We also computed the upper bound under the assumption that the return distribution is unbounded. As a reference point and for ease of comparisons, we report the Merton [1976] price, the jump-diffusion dynamic with diversified jump risk.

[Table 2.1 about here]

[Figure 2.1 about here]

The results presented in Table 2.1 show the jump-diffusion upper and lower bounds on the call options price. The maximum spread between the bounds is about 4.6% of the midpoint, comparable to the average bid-ask spread for at-the-money call options on the S&P 500 index. As expected from Proposition 2, equations (2.9)-(2.11), the upper bound is directly related to the diffusion risk premium and therefore the spread is an increasing function of $\mu - r$ while the lower bound is almost constant: unreported results show that the upper bound rises from 4.59 to 4.75 and to 4.91 for a risk premium equal to 4% and 6% respectively, while the lower bound stays approximately constant around 4.38. Unreported results also show that the bounds are much tighter for in-the-money options and the spread decreases to less than 2% for the base case. Similar unreported results show that the spread rises to 9.1% for the base case parameters when the options are 10% out-of-the-money. Note that the range of values of μ implies an ex-dividend risk premium range from 2% to 6%; a range that covers what most people would consider the appropriate value of such a premium in many important indexes. For the most commonly chosen risk premium of 4%, corresponding to $\mu = 6\%$, the spread of at-the-money options is about 8.1%. This is a tight bound if we consider the average bid-ask spread for at-the-money call options on the S&P 500 index. This range of allowable option prices in the stochastic dominance approach is the exact counterpart of the inability of the “traditional” arbitrage-based approaches to produce a single option price for jump diffusion processes without an arbitrarily chosen risk aversion parameter, even when the models have been augmented in this case by general equilibrium considerations. We further address this issue in the next section.

A major advantage of the stochastic dominance bounds in the jump-diffusion case is their relative insensitivity in the jump parameters, provided the total volatility is kept constant. Table 2.2 shows the value of the bounds for the ATM options for various values of the intensity parameter λ ranging from 0 to 1.9, with the total volatility $\sigma^2 + \lambda \left[(\mu_j - 0.5\sigma_j^2)^2 + \sigma_j^2 \right]$ kept constant to the base case value of 0.04444 by adjusting σ_j and the remaining parameters are kept constant as in the base case.¹⁴ As we can see, the bounds are tight and relatively insensitive to λ , while the spread decreases in λ from 5.24% to 4.1%. This weak dependence of the bounds on λ is particularly important, given the difficulty of estimating the parameters and the impossibility of estimating meaningful option prices by the “traditional” method for

¹⁴We discuss the choice of the range of intensity values in the next section.

all but the lowest values of the ranges of λ and the admissible risk aversion parameters.¹⁵

[Table 2.2 about here]

3 Equilibrium Analysis

In this section we consider the traditional approach to the extraction of the risk neutral distribution based on general equilibrium in the production economy, in which the underlying returns follow a jump-diffusion process, and compare its results to the stochastic dominance bounds of the previous section. Since the pricing kernel links the physical and risk-neutral densities in a general equilibrium setup, we derive the upper and lower bounds' pricing kernels in the SD approach, which are independent of investor's preferences with respect to rare events. We then use these kernels to restrict the preferences of the representative investor in the general equilibrium approach and extract appropriate bounds on the preference of the representative investor, which depend on option moneyness and time to maturity. Finally, we compare the SD implied bounds on the relative risk aversion (RRA) coefficient with those commonly used in the option pricing literature and those extracted from macroeconomic data, mostly based on joint consumptions and options data.

We summarize in our online appendix the general equilibrium analysis in production economy following a jump diffusion process as in (2.8), with a representative investor of the CRRA-type, with γ denoting the RRA coefficient. Of particular interest for our purposes are the expressions for the equilibrium pricing kernel π_t , its particular expression under the CRRA assumption, and the corresponding parameter mapping from the physical or P -distribution to the risk neutral Q -distribution. These mappings satisfy the requirements that $E_t [d(\pi_t S_t)] = 0$ ($\pi_t S_t$ should be a martingale), and $E_t [d\pi_t/\pi_t] = -r dt$. The derivation of the following expressions can be found in several studies, some of them derived under slightly more general conditions.¹⁶

The general expression for the pricing kernel process that satisfies the martingale restriction in a general equilibrium model with a CRRA representative investor follows the dynamics defined in equations (3.1) and (3.2).

$$d\pi_t/\pi_t = (-r - \lambda E [j_t^\pi - 1]) dt - \eta dW_t + (j_t^\pi - 1) dN_t, \quad (3.1)$$

where η , the risk premium of the diffusion component, is proportional to volatility and $j_t^\pi - 1$

¹⁵For instance, for a risk aversion coefficient of 7, the mid-range of the [Rosenberg and Engle \[2002\]](#) estimates, and for $\lambda = 10$ the total volatility of the option rises from 26.3% to 93% and becoming explosive on higher values of RRA and/or jump parameters.

¹⁶See, for instance, [Bates \[1991\]](#), [Bates \[2006\]](#), [Liu et al. \[2005\]](#) and [Zhang et al. \[2012\]](#).

is the relative jump amplitude of the pricing kernel process. In the particular case of the CRRA investor with RRA equal to γ we have

$$\eta = \gamma\sigma, \quad j_t^\pi = j_t^{-\gamma}. \quad (3.2)$$

Applying the definition of the pricing kernel on the basis of the martingale condition, the correspondence between the physical and risk neutral jump distribution parameters for the CRRA investor is found to be equal to

$$\begin{aligned} \lambda^Q &= \lambda E [j_t^{-\gamma}], \\ k^Q &= \frac{E [(j_t - 1) j_t^{-\gamma}]}{E [j_t^{-\gamma}]} \end{aligned} \quad (3.3)$$

Note also that in this model the total equilibrium risk premium must be equal to summation of the diffusive risk premium and the jump risk premium, $\mu_t - r = \gamma\sigma^2 + \lambda k - \lambda^Q k^Q$.

In the case of lognormal jump amplitude $\ln(j_t) \sim N[\mu_j - 0.5\sigma_j^2, \sigma_j^2]$, we have the following transformations.

$$\lambda^Q = \lambda \exp \left[-\gamma\mu_j + \frac{1}{2}\gamma(\gamma+1)\sigma_j^2 \right] \quad (3.4)$$

$$k^Q = E^Q [j_t^Q - 1] = \exp [\mu_j - \gamma\sigma_j^2] - 1 = \exp [\mu_j^Q] - 1 \quad (3.5)$$

With these relations the risk neutral jump diffusion dynamics become now

$$dS_t/S_t = \left[r - \lambda^Q E^Q [j_t^Q - 1] \right] dt + \sigma_t dW_t^Q + [j_t^Q - 1] dN_t^Q \quad (3.6)$$

Equations (3.1)-(3.6) summarize and describe completely the mapping from the P - to the risk neutral Q -distribution for a general equilibrium analysis of jump diffusion derivatives pricing given the existence of a representative CRRA investor, the only case that has appeared so far in the literature. The major drawback of this mapping is its dependence on the RRA parameter γ , for which there are widely differing estimates in the financial literature, reviewed further on in the next section. Since the stochastic dominance concept is much more general than equilibrium, we conclude now our theoretical analysis by deriving the limits on γ implied by the SD bounds of the previous section.

Embedding the SD bounds to an equilibrium model, we note that the pricing kernel equation

(3.1) should still hold, but the absence of a representative CRRA investor implies that (3.2) no longer holds. On the other hand, Propositions 2 and 3 introduce two risk-neutral distributions that yield the upper and lower option bounds when the underlying asset follows the jump-diffusion process. The violation of any of these two bounds implies that any trader can improve her utility by introducing the corresponding short or long option positions in her portfolio. A key factor in this estimation as compared to the equilibrium expressions is the fact that the total risk premium $\mu_t - r$ is an exogenous parameter, rather than an endogenously determined sum of the diffusive and jump risk premium equal to $\gamma\sigma^2 + \lambda k - \lambda^Q k^Q$ in the equilibrium model. Since utility maximization given the P -distribution (hence, the total risk premium) is a first step in the equilibrium approach, the SD bounds should be satisfied in this latter class of equilibrium models. The next result, part of which is obvious from (2.9)-(2.11) and (2.16)-(2.17) and the rest is proven in the appendix, helps establish bounds on the admissible equilibrium model values of γ given the P -distribution.

Proposition 4. *When the underlying asset follows a jump-diffusion process described by (3.6) the option bounds' corresponding risk neutral parameters are:*

For the upper bound:

$$\begin{aligned}\lambda^Q &= \lambda^U = \lambda E[j_t^\pi] = \lambda + \lambda_{Ut} = \lambda - \frac{\mu_t - r}{j_{\min} - 1}, \\ k^Q &= k^U = E^Q[j_t^Q - 1] = \frac{1}{E[j_t^\pi]} \times E[(j_t - 1) \times j_t^\pi] = \frac{\lambda}{\lambda + \lambda_{Ut}} k + \frac{\lambda_{Ut}}{\lambda + \lambda_{Ut}} (j_{\min} - 1).\end{aligned}\tag{3.7}$$

For the lower bound:

$$\begin{aligned}\lambda^Q &= \lambda^L = \lambda, \\ k^L &= E^Q[j_t^Q - 1] = \frac{1}{E[j_t^\pi]} \times E[(j_t - 1) \times j_t^\pi] = E(j_t - 1 | j \leq \bar{j}_t).\end{aligned}\tag{3.8}$$

If the jump amplitude is a truncated lognormal, the characteristic function of the jump component's distribution is $e^{\lambda T(f_j(\varphi) - 1)}$, where $f_j(\varphi) \equiv E(j^{i\varphi})$ is the characteristic function of the jump amplitude. In such a case the means and variances of the return distributions under the upper and lower bounds' Q -distributions are given by expressions (D.9)-(D.10) and (D.17)-(D.18) of the Appendix D and their truncated counterparts are given by (D.11)-(D.12) and (D.19)-(D.20). \square

In the next section we explore the equilibrium expressions summarized and/or derived in this section in order to find implicit bounds on the admissible values of the RRA parameter γ given the SD bounds defined on the basis of the independently estimated P -distribution parameters. The sources of these estimates include option pricing studies containing jump

diffusion and studies associated with the well-known equity premium puzzle, initially identified by [Mehra and Prescott \[1985\]](#).

4 RRA Values Implied by Stochastic Dominance

An exact expression giving the limits of the RRA compatible with the boundary risk neutral distributions of Propositions 2 and 3 is not available in closed form, especially in view of the fact that the transformed jump amplitudes are not lognormal. Such limits can only be defined numerically for a given set of parameters. In what follows we first find these limits for our base case and then examine several parameter values extracted from existing econometric studies of the S&P 500 returns' P -distribution. Figure 4.1 shows the admissible range of values of γ in the case of ATM options for our base case parameter values and for two alternative upper bounds, one based on the entire lognormal distribution $j_{min} = 0$ and the other on a lognormal distribution truncated at a worst-case return of -20% , i.e. $j_{min} = 0.8$. We find the implied RRA using the [Bates \[1991\]](#) jump-diffusion model to derive the equilibrium call option prices for a continuum of relative risk aversion coefficients up to 10.

[Figure 4.1 about here]

An alternative approximate interval of admissible values of γ can be found by assuming that the value of the option varies approximately with the return volatility, $Var^Q [dS/S]$, where the risk neutral distributions are given by (3.6), (2.9) and (2.16), respectively for the equilibrium, the upper bound, and the lower bound distributions. Since the diffusion component is the same for all three cases, this interval boils down to $Var [(j_t^L - 1) dN_t^L] \leq Var [(j_t^Q - 1) dN_t^Q] \leq Var [(j_t^U - 1) dN_t^U]$ with the jump distributions given by (3.4)-(3.5), (3.7) and (3.8) for the Q , U , and L cases respectively. Figure 4.2 shows the equilibrium implied jump variance for ATM options as a function of γ , together with its two limits, the P -distribution ($\gamma = 0$) and the upper bound.

[Figure 4.2 about here]

Both Figures 4.1 and 4.2 tell a consistent story. First of all, with respect to the SD lower bound, the only admissible value of a relative risk aversion coefficient for CRRA investor in Figure 4.1 is negative and equal to -1.72 for our base case, violating the risk aversion principle for the representative investor. This is not a surprising SD result, given that the bound lies below the Merton value, where the jump risk is systematic. The Merton prices are comparable to the [Bates \[1991\]](#) jump-diffusion prices if we assume that the representative investor is risk-neutral; that is the coefficient of the RRA is zero. More to the point, several econometric studies of S&P 500 index options based on the equilibrium approach and CRRA utilities have persistently documented negative values of γ , starting with [Jackw-](#)

erth [2000] and including Art-Sahalia and Lo [2000] and especially Ziegler [2007]. The latter study examined various potential explanations of this perverse result without reaching any definitive conclusion.¹⁷ The SD lower bound results are possible explanations of these negative γ findings, even though the implied pricing kernel is increasing. What they imply is that the equilibrium model cannot account for several risk neutral jump diffusion distributions compatible with the underlying P -distribution and the much weaker SD assumption of a declining pricing kernel. Since our purpose is the analysis of the admissible equilibrium model solutions within the SD framework, we shall ignore hereafter the SD lower bound and assume that the lowest SD-compatible value of γ is 0.

We now turn to the SD upper bound and restrict ourselves to the positive range of implied RRA. As illustrated in Figure 4.1, in our base case the maximum SD-admissible γ is 5.49 for the truncated lognormal, rising to almost 7 for the case of $j_{min} = 0$, under which there is a positive probability that the index will become worthless in the next 90 days. The implied γ is significantly larger on the basis of the volatility bounds, 6.59 for the truncated and 7.24 for the full lognormal jump amplitude. Note that, unlike the equilibrium model, the SD upper bound does not imply the same γ for all degrees of moneyness, as shown in Figure 4.3. Nonetheless, the range of upper bound-implied γ is relatively narrow, starting from 7.1 for 2% OTM up to 7.7 for 2% ITM. Unreported results show a similar narrow range of relative risk aversion also holds when the moneyness is kept constant but the time to expiration is varied from 0.083 to 1 year for the base case parameters.

[Figure 4.3 about here]

[Table 4.1 about here]

Table 4.1 shows the equilibrium option prices for our base case parameters and for the RRA range of Figures 4.1 to 4.3. The table also shows the corresponding implied mean μ and the risk neutral parameters of the jump component λ^Q and k^Q from (3.4)-(3.5) for the continuum of RRA.¹⁸ It is clear that the SD-restricted range of admissible RRA values needs to be tightened even further in order to accommodate reasonable values of the ex-dividend expected excess return, which is taken equal to 2% in our SD base case.

[Table 4.2 about here]

[Table 4.3 about here]

The SD implied upper bound on the relative risk aversion is consistent among a wide range of moneyness and time to maturity, as reported in Tables 4.2 and 4.3. Following the base

¹⁷The Ziegler [2007] studies potential explanations for the U-shaped and negative implied risk aversion patterns include (I) preference aggregation, both with and without stochastic volatility and jumps in returns, (II) misestimation of investors' beliefs caused by stochastic volatility, jumps, or a Peso problem, and (III) heterogeneous beliefs.

¹⁸Implied mean return is calculated based on the jump diffusion equilibrium risk premium, $\mu_t - r = \gamma\sigma^2 + \lambda k - \lambda^Q k^Q$, in Section 3.

case scenario, the 2% OTM call option reduces the SD implied upper bound on the RRA from 5.49 to 5.46 and the 2% ITM call option increases the upper bound on the RRA from 5.49 to 5.58. Similarly, as we increase time to maturity from one month to one year, the SD upper bounds on the call option prices increase, as expected, from 2.55 to 9.73, but the implied upper bound on the risk aversion increases correspondingly from 5.13 to 5.74. Hence, the implied upper bound on the RRA is relatively stable across a reasonable range of moneyness and time to maturity.

When the ex-dividend risk-premium increases from 2% to 4% we expect an increase in the SD upper bound on the RRA due to the direct relation between the SD option bounds and the mean μ of the returns process. Nonetheless, the relative insensitivity of the SD implied bound on RRA is still robust across the same range of moneyness and time to maturity, as shown in Tables 4.1 and 4.2 for the upper bound. As for the lower bound, the implied RRA is always zero based on the Merton jump-diffusion model irrespective of the level of risk premium and the option moneyness or time to maturity.

Since the SD-implied RRA is parameter dependent, we examine it for the parameter values that were estimated in earlier studies. Such studies fall into two categories, option market-based and macro-finance studies attempting to explain the equity premium puzzle. In empirical tests of the former category, a jump diffusion model is often included in a nested model that also includes stochastic volatility;¹⁹ only a few of these studies are reviewed here. [Bates \[1991\]](#) applied the nested models to Deutsche mark currency options, and in a subsequent study [Bates \[2000\]](#) to S&P 500 futures options, while [Pan \[2002\]](#) and [Rosenberg and Engle \[2002\]](#) examined S&P 500 index options, and [Bliss and Panigirtzoglou \[2004\]](#) FTSE 100 and S&P 500 index futures options. In these tests the parameters of the implied risk neutral distribution are extracted from cross sections of observed option prices and attempts are made to reconcile these option-based distributions with data from the market of the underlying asset. All studies stress the importance of jump risk premia in these reconciliation attempts.

Such reconciliations have not always been crowned with success, with the result that reported estimates of γ vary widely between studies. They range from an arbitrarily chosen value of 2 for [Bates \[1991\]](#) to 3.94 estimated by the same author in [Bates \[2006\]](#) using both return and option data, to a value up to 10 by [Liu and Pan \[2003\]](#), where they quantify the gain of including derivatives in portfolio optimization in the presence of jump. [Bliss and Panigirtzoglou \[2004\]](#) choose the risk aversion parameter between 3.37 and 9.52 to produce subjective densities that best fit the distributions of realized values. In a bootstrap estimate of the RRA based on observed 5-week S&P 500 options they report a minimum of -1.34 and a maximum of 8.17 for the relative risk aversion; note the approximate consistency of

¹⁹The equilibrium model does not allow stochastic volatility and jumps in linking the P - and Q -distributions. Although [Duffie et al. \[2000\]](#) have presented option prices under general Q -distributions containing both stochastic volatility and jumps, to our knowledge the only stochastic volatility pricing kernel was derived by [Christoffersen et al. \[2013\]](#) in the context of the [Heston \[1993\]](#). For stochastic volatility in the SD context see the Appendix E.

these varying estimates with the γ limits implied from our SD bounds shown in Figure 4.1 and Table 4.3 for the same maturity and a 4% risk premium. Liu et al. [2005] adjust the risk aversion coefficient to 3.49 to match an observed total equity premium when the underlying process follow jump-diffusion dynamic while the representative agent is averse not only to diffusive and jump risk but also to uncertainty aversion. However, as they point out, the data implied RRA coefficient has to be considerably larger than 3.49 if they only incorporate diffusive risk and jump risk in justifying the pronounced smirk pattern. More recently, Zhang et al. [2012] fit a jump-diffusion model with constant jump size to the underlying index data and show that a RRA coefficient of the order 2.134 is required to meet the observed risk-premium of the order 6%. However, they did not discuss if the proposed level of risk aversion is consistent with the observed option prices or even consistent with the jump risk premium required to match the observed premium.

Although the risk aversion values in these studies are mostly consistent with the SD implied bounds on RRA, the SD results are extracted uniquely from estimates of the underlying returns P -distribution. Compared to the equilibrium approach's estimates, they require an additional parameter, the total risk premium $\mu_t - r$, but do not require knowledge of γ . Unlike γ , there are reliable historical estimates of μ_t , even the largest of which defines tighter bounds on γ than those are available from empirical studies that rely on the option market. They can, therefore, verify the consistency of the two markets in a more reliable manner than the equilibrium approach. Note that the inconsistencies and inability of the equilibrium approach to reconcile the evidence of the underlying and option markets has already been mentioned in earlier studies.²⁰

A key issue in all the jump diffusion option pricing models is the accurate estimation of the parameters, since the Q -distributions for the option market fluctuate widely even for small differences in the parameter estimates. Further, the total risk premium does not appear explicitly and must be estimated from γ and the P -parameters, equal to $\gamma\sigma^2 + \lambda k - \lambda^Q k^Q$ as in (3.4) and (3.5). Since this premium is also a byproduct of the P -estimation, a successful reconciliation of the two markets must also verify the consistency of the premium with the value of γ used in the option market valuation. This is generally not done in most studies.

[Table 4.4 about here]

[Figure 4.4 about here]

We carry out this exercise for several econometric estimations of jump diffusion parameters shown in Table 4.4, whose results differ substantially not only between studies but also between differing data series within the same study. From the parameters estimate, we extract the appropriate RRA coefficient to match the reported P -distribution excess return in column 3 of Table 4.4. We find that γ should be below 2 in Andersen et al. [2002], and Eraker et al. [2003], and below 2.5 in Ramezani and Zeng [2007] and Honore [1998].

²⁰See Eraker et al. [2003, P. 1294], Broadie et al. [2007], Broadie et al. [2009], and more recently Ross [2015].

Therefore, none of the extracted underlying jump diffusion parameters can accommodate relative risk aversion coefficient above 2.5. Figure 4.4 shows the relationship between γ and the corresponding jump diffusion equilibrium risk premium $\gamma\sigma^2 + \lambda k - \lambda^Q k^Q$.

The SD implied bounds on the relative risk aversion can also provide information on the RRA coefficient extracted in macro finance studies. The RRA coefficients used in the option pricing literature are much lower than those of the equity premium puzzle studies, where [Mehra and Prescott \[1985\]](#) report a coefficient of 41, [Cochrane and Hansen \[1992\]](#) report RRA in the range of 40 – 50, and [Campbell and Cochrane \[1999\]](#) expects a value more than 35,²¹ although some argue that risk aversion this large implies implausible behavior along other dimensions;²² note that these studies relied on pure diffusion dynamics of consumption growth. Table 4.5 provides a partial explanation for this discrepancy.

[Table 4.5 about here]

In Table 4.5 we use the jump diffusion parameters of Table 4.4 to estimate the SD upper bound option prices (column 3) and then extract the implied upper bound relative risk aversion (column 4) by comparing these option upper bounds with the equilibrium option prices from the [Bates \[1991\]](#) model. Observe that for all of these parameter estimates the upper bound RRA values are similar to the ones found in the equity premium puzzle literature, the only option pricing model that can achieve such high γ values.

From the upper bound γ in column 4 we estimate the implied equity premium (column 5), using the [Mehra and Prescott \[1985\]](#) estimates. In the absence of rare events affecting consumption for CRRA investors, we have $\ln(E_t[R_{e,t+1}]) - \ln R_f = \gamma\sigma_{\Delta \ln C}^2$,²³ where the implied riskless rate R_f is found from the equation $\ln R_f = -\ln \beta + \gamma\mu_{\Delta \ln C} - 0.5\gamma^2\sigma_{\Delta \ln C}^2$, and where $\beta = 0.99$, $\mu_{\Delta \ln C} = 0.01919$ and $\sigma_{\Delta \ln C}^2 = 0.0011767$.²⁴ As we see, such a risk premium estimate is significantly lower than the observed risk premium (column 6) in four out of the six cases. This is, in fact, the equity premium puzzle as extensively addressed by different authors.²⁵

We now consider the equity premium puzzle from the SD perspective. Recall from the discrete time analysis that for any partition of the time to expiration, and by extension at the continuous time limit, the SD bounds are no arbitrage bounds, implying that if the option prices fail to lie between the bounds, any risk-averse investor can increase her expected utility by choosing a dominant portfolio containing the underlying, the riskless asset and a long or

²¹See also the survey article by [Kocherlakota \[1996\]](#)

²²See [Campanale et al. \[2010\]](#)

²³These estimates remained essentially unchanged when the data was extended to 2005 and then to 2009. See [Barro \[2006, Section 1.F\]](#) and [Backus et al. \[2011\]](#). This dataset has been used widely in most recent studies of the equity premium.

²⁴This is equivalent to the distribution of real consumer expenditure with mean of 0.02 and standard deviation of 0.035.

²⁵A good summary of the puzzle and its possible resolutions is in [Cochrane \[2001, Chapter 21\]](#), [Mehra and Prescott \[2003\]](#) and [Mehra \[2007\]](#). The expressions are in [Mehra and Prescott \[2003\]](#).

short option position in a conventional second-degree stochastic dominance comparison.²⁶ The SD assumptions then imply that there is at least one class of agents who increases their expected utility by such arbitrage trading. Since the SD upper bound gives the highest admissible option price implied by the P -distribution parameters, this price is equivalent to, ceteris paribus, the largest possible RRA coefficient compatible with the preferences of the representative option trader in an equilibrium model. Column 4 in Table 4.5 reports this upper bound RRA. Although we used the highest admissible risk aversion from an option trader’s perspective, columns 5 and 6 in the same table restate that we still observe the equity premium puzzle, as the corresponding equilibrium premium is lower than the observed one in most cases. Since the SD implied upper bound is a no arbitrage bound, one explanation for the above puzzle may be that index options are overpriced from the option trader’s perspective, as claimed in several empirical studies. Alternatively, as the option implied risk premium is forward looking, we can consider it as an alternative to the Ross [2015] Recovery Theorem.

Last, we explore the consistency of the upper bound-implied RRA with the results of more recent equity premium puzzle studies that consider the presence of fat tails in the consumption distribution.²⁷ In particular Barro [2006] has shown that rare disasters may account for high equity risk premia by using the international consumption dataset while maintaining a tractable framework of a representative agent with time-additive isoelastic preferences.²⁸ In his model the equity premium is given by $\ln(E_t[R_{e,t+1}]) - \ln R_f = \phi\gamma\sigma_{\Delta \ln C}^2 + \lambda E_t[(e^{-\gamma J} - 1)(1 - e^{\phi J})]$,²⁹ where $\phi = 1$ in this case and J is the amplitude of the consumption disaster risk, assumed lognormal, $\ln J \sim N(\mu_j, \sigma_j^2)$.

As a final exercise we apply the above equity premium equation using the upper bound RRA for the jump diffusion parameter estimates of Eraker et al. [2003], reported in Table 4.4. The implied equity premium in the presence of consumption disaster is 8.95, a level of premium that is above the observed 7.5% premium evaluated under the assumption that the risk free rate is 5%, but is close to the observed 8.5% premium if that rate is assumed to be a more realistic 4%.³⁰ Unfortunately, the implied equity premium is extremely sensitive to the jump parameter estimates.³¹ Considering the noticeable difference in equity premium based on the calibration assumed, it is difficult to make a conclusive decision regarding the implied equity premium in the presence of rare disasters unless there is a consensus opinion about

²⁶See Oancea and Perrakis [2014].

²⁷See, for instance, Barro [2006], Wachter [2013] and Martin [2013].

²⁸See Weitzman [2007], Jurek [2014], Barro and Ursua [2008], Gabaix [2008], Gourio [2008a], Gourio [2008b], Barro [2009], Gabaix [2012], and Wachter [2013] to name a few.

²⁹This is the continuous-time counterpart of Barro [2006] reproduced in Wachter [2013, Section I.G and Appendix C].

³⁰We assume that the consumption disaster has the mean, volatility, and intensity equal to 0.3, 0.15, and 0.01 respectively, following Backus et al. [2011].

³¹See the comments in Martin [2013, Section 2]. Similarly Ross [2015] describes the very low probability of catastrophic events and the extensive impact of minor changes in that perceived probability on asset prices as a dark matter of finance. For this reason we do not extend the exercise to the other jump diffusion estimates of Table 4.4.

the parameters of consumption's disaster distribution.

5 Extensions and Conclusions

The results presented in Section 2 yield bounds for jump-diffusion index option prices that are relatively simple to compute and reasonably tight for most empirically important cases. The alternative equilibrium approach that uses an assumed value of the relative risk aversion parameter to price the option implies a much wider and at times unrealistic range of admissible option prices if that parameter is allowed to vary over its relevant range, from 1 to more than 40 as implied by the equity premium puzzle studies. Further, the SD approach does not require the strong assumptions of the equilibrium approach such as the existence of a representative investor with constant RRA, even though it is capable of accommodating this case as well. In addition, the bounds can also accommodate state-dependent diffusion parameters, even though their computation would be difficult. Last but not least, the SD approach does not assume simultaneous equilibrium in the options and the underlying asset markets, an equilibrium that is not realistic if the options do not trade in an organized or a liquid market, as with catastrophe derivative instruments, where the instruments trade over the counter and the underlying process follows rare-event dynamics.³²

The discrete time approach of the bounds estimation allows several significant extensions to jump-diffusion option pricing. Thus, the valuation of American options is obvious, due to the discrete nature of the bounds. Second, the incorporation of proportional transaction costs is available for some (but not all) European or American option cases following the general results of [Constantinides and Perrakis \[2002\]](#) and [Constantinides and Perrakis \[2007\]](#); see, for instance, Proposition 1 in [Constantinides and Perrakis \[2002\]](#) for the upper bound of a call option. Last, extensions to the case of equity options are also feasible following an adaptation of the limiting SD approach for simple diffusion as in [Oancea and Perrakis \[2014\]](#).

³²See [Perrakis and Boloorforoosh \[2013\]](#).

Appendices

A Proof of Lemma 2

We prove the convergence of the discretization (2.6) in the i.i.d. case³³ where $\mu_t - \lambda k = \mu$, $\sigma_t = \sigma$, $j_t = j$. Convergence in the non-i.i.d. case follows from the convergence criteria for stochastic integrals, presented in Duffie and Protter [1992]. It is shown in an appendix, available from the authors on request.

The characteristic function of the terminal stock price at time T for a \$1 initial price under the jump-diffusion process (2.8) is

$$\begin{aligned}\varphi_{jD}(\omega) &= \exp\left[i\omega\mu T - \frac{\omega^2\sigma^2 T}{2}\right] \exp(-\lambda T) \sum_{N=0}^{\infty} \frac{(\lambda T)^N}{N!} [\varphi_j(\omega)]^N \\ &= \exp\left[i\omega\mu T - \frac{\omega^2\sigma^2 T}{2}\right] \exp[\lambda T(\varphi_j(\omega) - 1)],\end{aligned}\tag{A.1}$$

where $\varphi_j(\omega)$ is the characteristic function of the jump distribution. The first exponential corresponds to the diffusion component and the second to the jump component.

The characteristic function of the discretization (2.6) is

$$\varphi(\omega) = (\lambda\Delta t\varphi_j(\omega) + 1 - \lambda\Delta t) \left[\exp(i\omega\mu\Delta t)\varphi_\epsilon(\omega\sigma\sqrt{\Delta t}) \right],\tag{A.2}$$

where $\varphi_\epsilon(\omega)$ is the characteristic function of ϵ .³⁴ Since the distribution of ϵ has mean 0 and variance 1, we have

$$\begin{aligned}E[\epsilon] &= 0 = i\varphi'_\epsilon(0), \\ E[\epsilon^2] &= 1 = -\varphi''_\epsilon(0).\end{aligned}$$

By the Taylor expansion of $\varphi_\epsilon(\omega)$, we get

³³The proof is similar to that of Theorem 21.1 in Jacod and Protter [2003].

³⁴If instead of (2.6) we have a mixture of the diffusion and jump components then the characteristic function becomes $\varphi(\omega) = \lambda\Delta t\varphi_j(\omega) + (1 - \lambda\Delta t) \left[\exp(i\omega\mu\Delta t)\varphi_\epsilon(\omega\sigma\sqrt{\Delta t}) \right]$. The multiperiod convolution, however, still converges to (A.3).

$$\varphi(\omega) = (\lambda\Delta t\varphi_j(\omega) + 1 - \lambda\Delta t) \left[\exp(i\omega\mu\Delta t) \left[1 - \frac{\omega^2\sigma^2\Delta t}{2} + \omega^2\sigma^2\Delta t h(\omega\sigma\sqrt{\Delta t}) \right] \right],$$

where $h(\omega) \rightarrow 0$ as $\omega \rightarrow 0$. The multi-period convolution has the characteristic function $\varphi(\omega)^{(T/\Delta t)}$. Taking the limit, we have

$$\begin{aligned} \lim_{\Delta t \rightarrow 0} [\varphi(\omega)]^{T/\Delta t} &= \lim_{\Delta t \rightarrow 0} \exp \left[\frac{T}{\Delta t} \ln (\lambda\Delta t\varphi_j(\omega) + 1 - \lambda\Delta t) \right. \\ &\quad \left. + \frac{T}{\Delta t} \ln \left[\exp(i\omega\mu\Delta t) \left[1 - \frac{\omega^2\sigma^2\Delta t}{2} + \omega^2\sigma^2\Delta t h(\omega\sigma\sqrt{\Delta t}) \right] \right] \right] \quad (\text{A.3}) \\ &= \exp \left[\lambda T (\varphi_j(\omega) - 1) + i\omega\mu T - \frac{\omega^2\sigma^2 T}{2} \right] \end{aligned}$$

after applying *l'Hôpital's* rule. Equation (A.3) is, however, the same as equation (A.1), the characteristic function of (2.8). So, Levy's continuity theorem³⁵ proves the weak convergence of (2.6) to (2.8), QED. \square

Another way to characterize the limit process is its generator. Denote by $Z_{D,t}$ the diffusion component and by $Z_{j,t}$ the jump component of the return process. Therefore, we have

$$\begin{aligned} \lim_{\Delta t \rightarrow 0} \frac{E[f(S_{t+\Delta t}, t+\Delta t)] - f(S_t, t)}{\Delta t} &= \\ &= \lim_{\Delta t \rightarrow 0} \frac{E[f(S_{t(1+Z_{D,t+\Delta t}), t+\Delta t})] - f(S_t, t)}{\Delta t} + \lambda\Delta t \frac{E[f(S_{t(1+Z_{j,t+\Delta t}), t+\Delta t})] - f(S_t, t)}{\Delta t} \quad (\text{A.4}) \\ &= (\mu_t - \lambda k) S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma_t^2 S^2 \frac{\partial^2 f}{\partial S^2} + \lambda E[f(S_j) - f(S)], \end{aligned}$$

which gives us the generator of the price process described by (2.8), QED. \square

B Proof of Proposition 2

We follow the proof of Proposition 2 in [Oancea and Perrakis \[2014\]](#) and consider the same multiperiod discrete time option bounds, obtained by successive expectations under the risk-neutral upper bound distribution. We then seek the limit of this distribution as $\Delta t \rightarrow 0$. The multiperiod upper bound distribution is given by

³⁵See for instance [Jacod and Protter \[2003\]](#), Theorem 19.1].

$$U(z_{t+\Delta t}) = \begin{cases} P(z_{t+\Delta t} | S_t) & \text{with probability } \frac{R - z_{\min, t+\Delta t}}{E(z_{t+\Delta t}) - z_{\min, t+\Delta t}} \\ 1_{z_{\min, t+\Delta t}} & \text{with probability } \frac{E(z_{t+\Delta t}) - R}{E(z_{t+\Delta t}) - z_{\min, t+\Delta t}} \equiv Q \end{cases}, \quad (\text{B.1})$$

where $P(z_{t+\Delta t} | S_t)$ is the physical probability of return at each state at time $t + \Delta t$ and $1_{z_{\min, t+\Delta t}}$ is the physical probability for the lowest possible return. Assuming jump-diffusion dynamic as (2.8), the minimum outcome of the returns distribution is $j_{\min} - 1$, as discussed in Section 2. Since $z_{\min, t+\Delta t} = j_{\min} - 1$ the martingale transformation for the U -distribution clearly does not involve the diffusion component, which stays the same. The U -distribution is now a convolution of the diffusion component and a jump component with amplitude equal to $j_{\min} - 1$ and $j - 1$ with the probabilities of Q and $1 - Q$ respectively where Q is defined by the following equation.

$$\begin{aligned} Q &\equiv \frac{E(z_{t+\Delta t}) - R}{E(z_{t+\Delta t}) - z_{\min, t+\Delta t}} = \frac{E(z_{t+\Delta t}) - r\Delta t}{E(z_{t+\Delta t}) - (j_{\min} - 1)} \\ &= \frac{\mu_t \Delta t - r\Delta t}{\mu_t \Delta t - \sigma \max(|\epsilon|) \sqrt{\Delta t} - (j_{\min} - 1)} = -\frac{\mu_t - r}{(j_{\min} - 1)} \Delta t = \lambda_{Ut} \Delta t, \end{aligned} \quad (\text{B.2})$$

where λ_{Ut} is defined in Proposition 2.

Observe that λ_{Ut} is always positive since $(j_{\min} - 1) < 0$ and $E(z_{t+\Delta t}) > r\Delta t$. Hence, considering the multiperiod upper bound distribution (B.1) and equation (2.6), the discrete time upper bound process is as follows:

$$z_{t+\Delta t} = \begin{cases} z_{D, t+\Delta t} + (j - 1)\Delta N & \text{with probability } 1 - \lambda_{Ut} \Delta t \\ z_{D, t+\Delta t} + (j_{\min} - 1)\Delta N & \text{with probability } \lambda_{Ut} \Delta t \end{cases}. \quad (\text{B.3})$$

The outcomes of this process and their probabilities are as follows:

$$z_{t+\Delta t} = \begin{cases} z_{D, t+\Delta t} & \text{with probability } (1 - \lambda \Delta t)(1 - \lambda_{Ut} \Delta t) \\ z_{D, t+\Delta t} + (j - 1) & \text{with probability } \lambda \Delta t(1 - \lambda_{Ut} \Delta t) \\ z_{D, t+\Delta t} + (j_{\min} - 1) & \text{with probability } \lambda_{Ut} \Delta t \end{cases}. \quad (\text{B.4})$$

By removing the terms in $o(\Delta t)$, the upper bound process outcomes become

$$z_{t+\Delta t} = \begin{cases} z_{D, t+\Delta t} & \text{with probability } 1 - (\lambda + \lambda_{Ut}) \Delta t \\ z_{D, t+\Delta t} + (j_t^U - 1) & \text{with probability } (\lambda + \lambda_{Ut}) \Delta t \end{cases}. \quad (\text{B.5})$$

where j_t^U is given by (2.11). This process, however, corresponds to (2.9), QED. \square

The generator of the price process, which is also reflected in equation (2.12), is

$$A^U f = \frac{1}{2} \sigma_t^2 S^2 \frac{\partial^2 f}{\partial S^2} + [r - (\lambda + \lambda_{U_t}) k^U] S \frac{\partial f}{\partial S} + \frac{\partial f}{\partial T} + (\lambda + \lambda_{U_t}) E^U [f(S j_t^U) - f(S)] \quad (\text{B.6})$$

,QED. □

C Proof of Proposition 3

The proof is very similar to those of Lemma 2 and Proposition 2. Assuming, for simplicity, that both ϵ and j have continuous distributions, we may apply the multiperiod lower bound distribution, given by

$$L(z_{t+\Delta t}) = P(z_{t+\Delta t} | S_t, z_{t+\Delta t} \leq z_t^*) \quad \text{such that} \quad E(z_{t+\Delta t} | S_t, z_{t+\Delta t} \leq z_t^*) = R. \quad (\text{C.1})$$

From the convergence of return process without the jump component to the diffusion process,³⁶ it is clear that as $\Delta t \rightarrow 0$ all the outcomes of the diffusion component will be lower than \bar{j}_t . Therefore, the limiting distribution will include the whole diffusion component and a truncated jump component. The maximum jump outcome in this truncated distribution is obtained from the condition that the distribution is risk neutral, which is expressed in (2.17). We observe that the lower bound distribution over $(t, t + \Delta t)$ is the sum of the diffusion component and a jump of intensity λ and log-amplitude distribution j_t^L , the truncated distribution $\{j | j \leq \bar{j}_t\}$.

$$z_{t+\Delta t} = \begin{cases} z_{D,t+\Delta t} & \text{with probability } 1 - \lambda \Delta t \\ z_{D,t+\Delta t} + (j_t^L - 1) \Delta N & \text{with probability } \lambda \Delta t \end{cases} \quad (\text{C.2})$$

By Lemma 2 this process converges weakly for to the jump-diffusion process (2.16), QED.□

The generator of the price process is

$$A^L f = \frac{1}{2} \sigma_t^2 S^2 \frac{\partial^2 f}{\partial S^2} + [r - \lambda k^L] S \frac{\partial f}{\partial S} + \frac{\partial f}{\partial T} + \lambda E^L [f(S j_t^L) - f(S)] \quad (\text{C.3})$$

³⁶More detail can be find in [Oancea and Perrakis \[2014\]](#) and [Merton \[1992\]](#)

which appears in equation (2.18), QED. □

D Characteristic Function and Moments of Returns Dynamic

When the underlying process under P is defined by equation (2.8), then log return process is

$$\begin{aligned} \ln(S_t/S_0) &= \left[\left(\mu - \frac{1}{2}\sigma^2 - \lambda k \right) t + \sigma W_t + \sum_{i=1}^{N_t} J_i \right] \\ &= \left[\left(\mu - \frac{1}{2}\sigma^2 - \lambda k \right) t + \sigma W_t + \sum_{i=1}^{N_t} \ln(j_i) \right] \end{aligned} \quad (\text{D.1})$$

The characteristic function of the log return process can be defined as the following expectation or simply by the Fourier transform of log-return density function.

$$\begin{aligned} f_\varphi(\ln(S_t/S_0)) &= E[\exp(i\varphi \ln(S_t/S_0))] \\ &= E\left[\exp\left[i\varphi\left(\mu - \frac{1}{2}\sigma^2 - \lambda k\right)t\right]\right] E[\exp(i\varphi\sigma W_t)] E\left[\exp\left(\sum_{i=1}^{N_t} i\varphi J_i\right)\right] \\ &= \exp\left[i\varphi\left(\mu - \frac{1}{2}\sigma^2 - \lambda k\right)t\right] \exp\left[\frac{1}{2}(i\varphi\sigma)^2 t\right] E\left[\exp\left(\sum_{i=1}^{N_t} i\varphi \ln(j_i)\right)\right] \\ &= \exp\left[i\varphi\left(\mu - \frac{1}{2}\sigma^2 - \lambda k\right)t - \frac{1}{2}\varphi^2\sigma^2 t\right] [\exp(\lambda t E(j^{i\varphi} - 1))] \\ &= \exp\left[i\varphi\mu t - \frac{1}{2}i\varphi\sigma^2 t - i\varphi\lambda k t - \frac{1}{2}\varphi^2\sigma^2 t + \lambda t E(j^{i\varphi} - 1)\right] \\ f_\varphi(\ln(S_t/S_0)) &= \exp\left[i\varphi\mu t - \frac{1}{2}i\varphi(1 - i\varphi)\sigma^2 t + \lambda[E(j^{i\varphi} - 1) - i\varphi k]t\right] \end{aligned} \quad (\text{D.2})$$

The second line is based on a pdf of Poisson counter using the property of law of iterated expectation and the third line is based on the Taylor expansion of exponential function. Note that all j_i are identically distributed as j . Expectation of $E(j^{i\varphi} - 1)$ is also defined by the law of iterated expectations. Using the above characteristic function, the mean and the volatility of the log return process can be defined with the derivatives of the characteristic

function.

$$E \left[\ln(S_t/S_0) \right] = (-i) \frac{\partial f}{\partial \varphi} \Big|_{\varphi=0} = \left(\mu - \frac{1}{2} \sigma^2 + \lambda E[\ln j] - \lambda k \right) t$$

$$Var \left[\ln(S_t/S_0) \right] = (-i)^2 \frac{\partial^2 f}{\partial \varphi^2} \Big|_{\varphi=0} = \left(\sigma^2 + \lambda (E[\ln j])^2 + \lambda (Var[\ln j]) \right) t$$

When the jump size is log normal, $Ln(j) \sim N(\mu_j - \frac{1}{2}\sigma_j^2, \sigma_j^2)$ or $j \sim LogN(e^{\mu_j}, e^{2\mu_j}(e^{\sigma_j} - 1))$,

$$E[\ln(S_t/S_0)] = \mu t - \frac{1}{2} \sigma^2 t + \lambda (\mu_j - \frac{1}{2} \sigma_j^2) t - \lambda k t \quad (D.3)$$

$$Var[\ln(S_t/S_0)] = \sigma^2 t + \lambda \left[(\mu_j - \frac{1}{2} \sigma_j^2)^2 + \sigma_j^2 \right] t \quad (D.4)$$

In the case of risk-neutral process $J^Q = \ln(j^Q) \sim N(\mu_j - \gamma \sigma_j^2 - \frac{1}{2} \sigma_j^2, \sigma_j^2)$

$$f_\varphi^Q \left[\ln \frac{S_t}{S_0} \right] = \exp \left[i\varphi r t - \frac{1}{2} i\varphi (1 - i\varphi) \sigma^2 t + \lambda^Q t [E^Q(j^{i\varphi} - 1) - i\varphi k^Q] \right] \quad (D.5)$$

$$E^Q[\ln(S_t/S_0)] = r t - \frac{1}{2} \sigma^2 t + \lambda^Q [\mu_j - \gamma \sigma_j^2 - \frac{1}{2} \sigma_j^2] t - \lambda^Q k^Q t \quad (D.6)$$

$$Var^Q[\ln(S_t/S_0)] = \sigma^2 t + \lambda^Q \left[(\mu_j - \gamma \sigma_j^2 - \frac{1}{2} \sigma_j^2)^2 + \sigma_j^2 \right] t \quad (D.7)$$

Following the Proposition 2, when the underlying asset follows the dynamic of (2.9), the upper bound characteristic function and its first two central moments can be defined similarly by equations (D.8), (D.9), and (D.10) respectively.

$$f_\varphi^U \left[\ln \frac{S_t}{S_0} \right] = \exp \left[i\varphi r t - \frac{1}{2} i\varphi (1 - i\varphi) \sigma^2 t + (\lambda + \lambda_{Ut}) [E^U(j^{i\varphi} - 1) - i\varphi k^U] t \right] \quad (D.8)$$

$$E^U \left[\ln \frac{S_t}{S_0} \right] = r t - \frac{1}{2} \sigma^2 t + (\lambda + \lambda_{Ut}) E^U[\ln(j^U)] t - (\lambda + \lambda_{Ut}) k^U t$$

$$= r t - \frac{1}{2} \sigma^2 t + \lambda (\mu_j - \frac{1}{2} \sigma_j^2) t + \lambda_{Ut} (\ln j_{\min}) t - (\lambda + \lambda_{Ut}) k^U t \quad (D.9)$$

$$\begin{aligned}
Var^U \left[\ln \frac{S_t}{S_0} \right] &= \sigma^2 t + (\lambda + \lambda_{Ut}) (E^U [\ln(j^U)])^2 t + (\lambda + \lambda_{Ut}) (Var^U [\ln(j^U)]) t \\
&= \sigma^2 t + \frac{1}{\lambda + \lambda_{Ut}} \left[\lambda \left(\mu_j - \frac{1}{2} \sigma_j^2 \right) + \lambda_{Ut} (\ln(j_{\min})) \right]^2 t + \lambda \sigma_j^2 t
\end{aligned} \tag{D.10}$$

In the analysis of the upper bound, we discuss the limiting distribution that includes the diffusion component and a truncated jump component where the truncation limit is chosen to meet the observed jump amplitude in econometric studies of jump diffusion.³⁷ In this case the first and second central moments can be defined by equations (D.11) and (D.12) where Φ is the Normal cumulative function and ϕ is the Normal probability function.

$$\begin{aligned}
E^U \left[\ln \frac{S_t}{S_0} \mid j > j_{\min} \right] &= rt - \frac{1}{2} \sigma^2 t \\
&+ \left[\lambda \left(\mu_j - \frac{1}{2} \sigma_j^2 \right) + \lambda_{Ut} (\ln j_{\min}) + \sqrt{\lambda(\lambda + \lambda_{Ut})} \sigma_j \frac{\phi(a_0)}{1 - \Phi(a_0)} \right] t \\
&- (\lambda + \lambda_{Ut}) k^U \times \frac{\Phi(\sigma_j - a_0)}{\Phi(-a_0)} t
\end{aligned} \tag{D.11}$$

$$\begin{aligned}
Var^U \left[\ln \frac{S_t}{S_0} \mid j > j_{\min} \right] &= \sigma^2 t + (\lambda + \lambda_{Ut}) \times \\
&\left[\frac{\lambda}{\lambda + \lambda_{Ut}} \left(\mu_j - \frac{1}{2} \sigma_j^2 \right) + \frac{\lambda_{Ut}}{\lambda + \lambda_{Ut}} (\ln j_{\min}) + \sqrt{\frac{\lambda}{\lambda + \lambda_{Ut}}} \sigma_j \frac{\phi(a_0)}{1 - \Phi(a_0)} \right]^2 t \\
&+ \lambda \sigma_j^2 \left[1 + \frac{a_0 \phi(a_0)}{1 - \Phi(a_0)} - \left(\frac{\phi(a_0)}{1 - \Phi(a_0)} \right)^2 \right]
\end{aligned} \tag{D.12}$$

where $a_0 = [\ln(j_{\min}) - (\mu_j - 0.5 \times \sigma_j^2)] / \sigma_j$.

Another important special case discussed in Remark 2.4 and equation (2.13) where the lower limit of the jump amplitude is equal to 0. Therefore, $j_{\min} = 0$ and the return distribution has an absorbing state in which the stock becomes worthless. In this case the upper bound characteristic function and its central moments are as follow.

$$f_\varphi^{j_{\min}=-1} \left[\ln \frac{S_t}{S_0} \right] = \exp \left[i\varphi \mu t - \frac{1}{2} i\varphi (1 - i\varphi) \sigma^2 t + \lambda [E(j^{i\varphi}) - 1] - i\varphi k \right] t \tag{D.13}$$

³⁷See Lien [1985] regarding truncated lognormal distributions.

$$E^{j_{\min}=-1} [\ln(S_t/S_0)] = \mu t - \frac{1}{2}\sigma^2 t + \lambda(\mu_j - \frac{1}{2}\sigma_j^2)t - \lambda k t \quad (\text{D.14})$$

$$Var^{j_{\min}=-1} [\ln(S_t/S_0)] = \sigma^2 t + \lambda \left[(\mu_j - \frac{1}{2}\sigma_j^2)^2 + \sigma_j^2 \right] t \quad (\text{D.15})$$

Similarly, we introduce the lower bound characteristic function and its central moments when the underlying asset follows the dynamic of (2.16), as in Proposition 3.

$$f_\varphi^L \left[\ln \frac{S_t}{S_0} \right] = \exp \left[i\varphi r t - \frac{1}{2}i\varphi(1-i\varphi)\sigma^2 t + \lambda [E^L(j^{i\varphi} - 1) - i\varphi k^L] t \right] \quad (\text{D.16})$$

$$E^L [\ln(S_t/S_0)] = r t - \frac{1}{2}\sigma^2 t + \lambda E^L [\ln(j^L)] t - \lambda k^L t \quad (\text{D.17})$$

$$Var^L [\ln(S_t/S_0)] = \sigma^2 t + \lambda (E^L [\ln(j^L)])^2 t + \lambda (Var^L [\ln(j^L)]) t \quad (\text{D.18})$$

Accordingly, if the distribution of $J = \ln(j)$ is normal and truncated at the upper bound $\ln(\bar{j})$ then the central moments are given by (D.19) and (D.20) where $b_0 = [\ln(\bar{j}) - (\mu_j - 0.5\sigma_j^2)]/\sigma_j$.

$$\begin{aligned} E^L \left[\ln \frac{S_t}{S_0} \mid j < \bar{j} \right] &= r t - \frac{1}{2}\sigma^2 t + \lambda E^L [\ln(j^L) \mid j < \bar{j}] - \lambda E^L [j^L - 1 \mid j < \bar{j}] t \\ &= r t - \frac{1}{2}\sigma^2 t + \lambda \left[(\mu_j - \frac{1}{2}\sigma_j^2) + \sigma_j \frac{\phi(b_0)}{\Phi(b_0)} \right] \\ &\quad - \lambda k \times \frac{\Phi(-\sigma_j + b_0)}{\Phi(b_0)} t \end{aligned} \quad (\text{D.19})$$

$$\begin{aligned} Var^L \left[\ln \frac{S_t}{S_0} \mid j < \bar{j} \right] &= \sigma^2 t + \lambda \left[E^L [\ln(j^L) \mid j < \bar{j}] \right]^2 t + \lambda [Var^L [\ln(j^L) \mid j < \bar{j}]] t \\ &= \sigma^2 t + \lambda \left[\mu_j - \frac{1}{2}\sigma_j^2 + \sigma_j \frac{\phi(b_0)}{\Phi(b_0)} \right]^2 t \\ &\quad + \lambda \sigma_j^2 \left[1 - \frac{b_0 \phi(b_0)}{\Phi(b_0)} - \left(\frac{\phi(b_0)}{\Phi(b_0)} \right)^2 \right] t \end{aligned} \quad (\text{D.20})$$

E Stochastic Volatility and Jumps Under Stochastic Dominance

Here we discuss how the incorporation of stochastic volatility (SV) will affect the jump diffusion SD bounds on index options. SV introduces an additional source of systematic risk, which can be handled either by arbitrage or by equilibrium considerations. We sketch below an extension of our approach to the pricing of jump risk that can incorporate SV, provided its systematic risk implications are handled outside our model.

In a combined SV and jump-diffusion process, the stock returns are still given by (2.8) but the volatility σ_t is random and follows a general diffusion, often a mean-reverting Ornstein-Uhlenbeck process.³⁸ In our case we use a general form with an unspecified instantaneous mean $m(\sigma_t^2)$ and volatility $s(\sigma_t^2)$. The asset dynamics then become

$$\begin{aligned} dS_t/S_t &= (\mu_t - \lambda k)dt + \sigma_t dW_1 + (j - 1)dN \\ d\sigma_t^2 &= m(\sigma_t^2)dt + s(\sigma_t^2)dW_2, \end{aligned} \tag{E.1}$$

where the two Brownian motions are correlated as $dW_1 \cdot dW_2 = \rho \sigma_t^2 dt$. The following discrete representation (E.2) can be easily shown by applying Lemma 2 to converge to (E.1):³⁹

$$\begin{aligned} (S_{t+\Delta t} - S_t)/S_t &\equiv z_{t+\Delta t} = \mu(S_t)\Delta t + \sigma_t \epsilon \sqrt{\Delta t} + (j - 1)\Delta N \\ \sigma_{t+\Delta t}^2 - \sigma_t^2 &= m(\sigma_t^2)\Delta t + s(\sigma_t^2)\varsigma \sqrt{\Delta t} \end{aligned} \tag{E.2}$$

Where ς is an error term of mean 0 and variance 1, and with correlation $\rho(\sigma_t^2)$ between ϵ and ς . In what follows we shall assume that this correlation is constant.

Under reasonable regularity conditions the pricing kernel at time t conditional on the state variable vector (S_t, σ_t) is monotone decreasing. Similarly, for any given σ_t the option price is convex in the stock price.⁴⁰ Hence, for any given volatility path over the interval $[0, T]$ to option expiration the option prices at any time t are bound by the expressions $\overline{C}_t(S_t, \sigma_t)$ and $\underline{C}_t(S_t, \sigma_t)$ given in (2.3). Since both of these expressions are expected option payoffs under risk neutral distributions, we can apply arbitrage methods as in Merton [1976] to price the options given a price $\xi(S_t, \sigma_t, t)$ for the volatility risk. Proposition 2 and 3, therefore, hold and the admissible option's upper bound satisfies the PDE in (E.3) and its lower counterpart satisfies the PDE in equation (E.4).

³⁸See Heston [1993].

³⁹We also use the proof of the convergence of the diffusion process discussed in Oancea and Perrakis [2014]. In the extension of the proof to stochastic volatility, the only difference is related to the vector ϕ_t in applying the Lindeberg condition, which is now a two-dimensional (S_t, σ_t^2) vector.

⁴⁰See the results of Bergman et al. [1996].

$$\begin{aligned}
& \frac{1}{2}\sigma_t^2 S^2 \frac{\partial^2 \bar{C}}{\partial S^2} + [r - (\lambda + \lambda_{U_t})k^U] S \frac{\partial \bar{C}}{\partial S} + \rho \sigma_t s(\sigma_t^2) \frac{\partial^2 \bar{C}}{\partial S \partial \sigma_t^2} + \frac{1}{2}s^2(\sigma_t^2) \frac{\partial^2 \bar{C}}{\partial^2 \sigma_t^2} + \\
& + [m(\sigma_t^2) - \xi(S_t, \sigma_t, t)] \frac{\partial \bar{C}}{\partial \sigma_t^2} - \frac{\partial \bar{C}}{\partial T} + (\lambda + \lambda_{U_t})E^U [\bar{C}(Sj_t^U) - \bar{C}(S)] = r\bar{C}
\end{aligned} \tag{E.3}$$

$$\begin{aligned}
& \frac{1}{2}\sigma_t^2 S^2 \frac{\partial^2 \underline{C}}{\partial S^2} + [r - \lambda k^L] S \frac{\partial \underline{C}}{\partial S} + \rho \sigma_t s(\sigma_t^2) \frac{\partial^2 \underline{C}}{\partial S \partial \sigma_t^2} + \frac{1}{2}s^2(\sigma_t^2) \frac{\partial^2 \underline{C}}{\partial^2 \sigma_t^2} + \\
& + [m(\sigma_t^2) - \xi(S_t, \sigma_t, t)] \frac{\partial \underline{C}}{\partial \sigma_t^2} - \frac{\partial \underline{C}}{\partial T} + \lambda E^L [\underline{C}(Sj_t^L) - \underline{C}(S)] = r\underline{C}
\end{aligned} \tag{E.4}$$

The estimation of (E.3)-(E.4) under general conditions presents computational challenges that lie outside the scope of this paper and remains a topic for future research.

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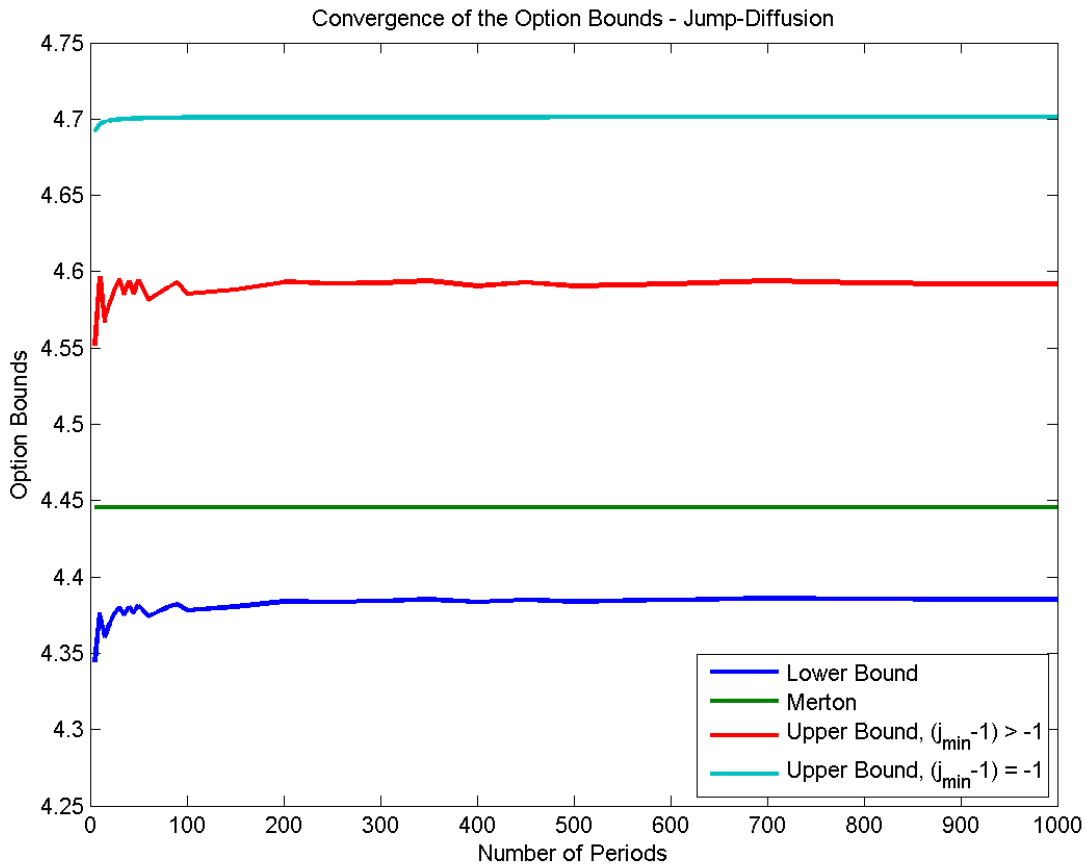


Figure 2.1: Convergence of Jump-Diffusion Option Bounds

This figure illustrates the convergence of the option bounds under a jump-diffusion process for an ATM option with $X = 100$, time to maturity $T = 0.25$ years, and with the following annual parameters: $r = 2\%$, $\mu = 4\%$, $\sigma = 20\%$, $\lambda = 0.6$, $\mu_j = -0.05$, $\sigma_j = 7\%$. The jump size distribution is lognormal. In the case $j_{min} - 1 > -1$, the distribution was truncated to a worst case jump return of -20% . When $j_{min} - 1 = -1$, the return distribution has an absorbing state where the stock becomes worthless.

Table 2.1

The table shows the convergence of the jump-diffusion bounds for an ATM option with $X = 100$ and time to maturity $T = 0.25$ years with $r = 2\%$, $\mu = 4\%$, $\sigma = 20\%$, $\lambda = 0.6$, $\mu_j = -0.05$, $\sigma_j = 7\%$, annual parameters. The jump amplitude distribution is lognormal. In the case $j_{min} - 1 > 1$, the distribution was truncated to a worst-case jump return of -20% . In the last column we present the case when the lower limit of the jump amplitude is equal to 0, in which $j_{min} - 1 = 1$, that is the return distribution has an absorbing state where the stock becomes worthless.

Periods	Lower Bound	Merton Price	Upper	Upper
			Bound	Bound
			$(j_{min} - 1 > -1)$	$(j_{min} - 1 = -1)$
5	4.3443	4.4455	4.5521	4.6920
10	4.3764	4.4455	4.5972	4.6964
15	4.3606	4.4455	4.5671	4.6983
20	4.3694	4.4455	4.5784	4.6990
25	4.3757	4.4455	4.5878	4.6994
30	4.3800	4.4455	4.5955	4.6996
35	4.3752	4.4455	4.5851	4.7000
40	4.3802	4.4455	4.5938	4.7001
45	4.3763	4.4455	4.5858	4.7003
50	4.3811	4.4455	4.5943	4.7003
60	4.3743	4.4455	4.5815	4.7006
70	4.3772	4.4455	4.5854	4.7007
80	4.3797	4.4455	4.5892	4.7007
90	4.3820	4.4455	4.5929	4.7008
100	4.3781	4.4455	4.5856	4.7009
150	4.3804	4.4455	4.5881	4.7010
200	4.3838	4.4455	4.5931	4.7011
250	4.3837	4.4455	4.5922	4.7011
300	4.3842	4.4455	4.5927	4.7011
350	4.3851	4.4455	4.5939	4.7012
400	4.3834	4.4455	4.5904	4.7012
450	4.3849	4.4455	4.5928	4.7012
500	4.3838	4.4455	4.5906	4.7012
600	4.3848	4.4455	4.5920	4.7012
700	4.3860	4.4455	4.5940	4.7012
800	4.3855	4.4455	4.5926	4.7013
900	4.3852	4.4455	4.5920	4.7013
1,000	4.3852	4.4455	4.5918	4.7013

Table 2.2

The table shows the jump-diffusion bounds for an ATM option with $X = 100$ and time to maturity $T = 0.25$ years, and annual parameters $r = 2\%$, $\mu = 4\%$, $\sigma = 20\%$, $\mu_j = -0.05$, for various values of the intensity parameter and the jump amplitude volatility σ_j . We vary the jump volatility and intensity, keeping the overall volatility of the jump-diffusion constant equal to 0.04444. The jump amplitude distribution is lognormal. In the case, $j_{min} - 1 > 1$ the distribution was truncated to a worst-case jump return of -20% .

Lambda	Jump Vol. (σ_j)	Lower Bound	Merton Price	Upper Bound ($j_{min} - 1 > -1$)	Upper Bound ($j_{min} - 1 = -1$)
0.0	0.0000	4.2275	4.2312	4.2348	4.4842
0.1	0.1996	4.2601	4.3417	4.4832	4.5966
0.2	0.1377	4.2927	4.3991	4.5332	4.6548
0.3	0.1093	4.3226	4.4253	4.5603	4.6813
0.4	0.0918	4.3482	4.4364	4.5755	4.6924
0.5	0.0794	4.3689	4.4417	4.5849	4.6976
0.6	0.0700	4.3852	4.4455	4.5918	4.7013
0.7	0.0624	4.3977	4.4491	4.5973	4.7048
0.8	0.0560	4.4074	4.4529	4.6020	4.7086
0.9	0.0505	4.4151	4.4568	4.6061	4.7124
1.0	0.0456	4.4214	4.4606	4.6099	4.7161
1.1	0.0412	4.4267	4.4643	4.6134	4.7197
1.2	0.0371	4.4314	4.4679	4.6166	4.7231
1.3	0.0332	4.4356	4.4713	4.6198	4.7264
1.4	0.0295	4.4393	4.4745	4.6228	4.7296
1.5	0.0258	4.4427	4.4777	4.6257	4.7326
1.6	0.0221	4.4457	4.4808	4.6285	4.7356
1.7	0.0183	4.4486	4.4838	4.6313	4.7385
1.8	0.0140	4.4514	4.4868	4.6340	4.7413
1.9	0.0085	4.4541	4.4897	4.6366	4.7441

Table 4.1

This table shows the sensitivity of the equilibrium jump-diffusion call option prices to the coefficient of relative risk aversion γ for a continuum of coefficients up to 40. The base case parameters are $S = 100$, $X = 100$, $T = 0.25$, $r = 2\%$, $\mu = 4\%$, $\sigma = 20\%$, $\lambda = 0.6$, $\mu_j = -0.05$, $\sigma_j = 7\%$. The call option prices are based on the [Bates \[1991\]](#) jump-diffusion model. Implied mean return is calculated based on the jump diffusion equilibrium risk premium in Section 3.

Risk Aversion	Call Price	Implied Mean	Risk Neutral Jump Intensity	Risk Neutral Jump Size
-2.00	4.3846	-0.0676	0.5482	-0.0370
-1.00	4.4007	-0.0240	0.5721	-0.0418
0.00	4.4198	0.0200	0.6000	-0.0464
0.25	4.4251	0.0311	0.6076	-0.0476
0.50	4.4307	0.0422	0.6156	-0.0488
0.75	4.4365	0.0533	0.6238	-0.0499
1.00	4.4425	0.0644	0.6323	-0.0511
1.25	4.4489	0.0756	0.6411	-0.0523
1.50	4.4554	0.0869	0.6503	-0.0534
1.75	4.4623	0.0981	0.6598	-0.0546
2.00	4.4694	0.1095	0.6696	-0.0557
2.25	4.4769	0.1208	0.6798	-0.0569
2.50	4.4847	0.1322	0.6904	-0.0580
2.75	4.4928	0.1437	0.7013	-0.0592
3.00	4.5012	0.1551	0.7126	-0.0604
3.25	4.5101	0.1667	0.7244	-0.0615
4.00	4.5388	0.2016	0.7621	-0.0649
6.00	4.6359	0.2977	0.8846	-0.0741
8.00	4.7723	0.3991	1.0470	-0.0831
10.00	4.9648	0.5085	1.2639	-0.0920
15.00	5.8776	0.8435	2.2043	-0.1140
20.00	7.9741	1.3808	4.3456	-0.1355
25.00	12.2467	2.5064	9.6835	-0.1564
30.00	20.7023	5.5042	24.3900	-0.1768
40.00	65.6746	49.8924	223.4470	-0.2162

Table 4.2

This table shows the sensitivity of the SD implied upper bound relative risk aversion to the moneyness and unconditional mean return. SD upper bound on call option prices are calculated for (columns 3 and 6) and for $j_{min} - 1 = -0.8$ the full support of jump distribution (columns 4 and 8). The parameters are $S = 100$, $T = 0.25$, $r = 2\%$, $\mu = 4\% - 6\%$, $\sigma = 20\%$, $\lambda = 0.6$, $\mu_j = -0.05$, $\sigma_j = 7\%$.

Money ness (X/S)	2% Risk Premium			4% Risk Premium		
	Implied Relative Risk Aversion	Upper Bound (Truncated)	Upper Bound	Implied Relative Risk Aversion	Upper Bound (Truncated)	Upper Bound
0.95	8.36	7.5917	7.7691	11.4630	7.77	8.1059
0.96	8.13	6.9206	7.0831	11.2042	7.10	7.4072
0.97	7.92	6.2842	6.4325	10.9652	6.46	6.7427
0.98	7.72	5.6835	5.8182	10.7437	5.86	6.1137
0.99	7.54	5.1193	5.2410	10.5376	5.29	5.5210
1.00	7.37	4.5918	4.7013	10.3442	4.75	4.9653
1.01	7.24	4.1030	4.2011	10.1802	4.26	4.4487
1.02	7.11	3.6508	3.7384	10.0276	3.80	3.9693
1.03	6.99	3.2345	3.3122	9.8835	3.38	3.5265
1.04	6.87	2.8530	2.9216	9.7443	2.99	3.1194
1.05	6.77	2.5060	2.5664	9.6189	2.63	2.7480

Table 4.3

This table shows the sensitivity of the SD implied upper bound relative risk aversion to the time to maturity of the options from one-month expiration until one year to expiration and unconditional mean return. The SD upper bound on the call option prices is for the whole support of jump distribution. The base case parameters are $S = 100$, $X = 100$, $r = 2\%$, $\mu = 4\% - 6\%$, $\sigma = 20\%$, $\lambda = 0.6$, $\mu_j = -0.05$, $\sigma_j = 7\%$.

Time to Maturity	2% Risk Premium		4% Risk Premium	
	Implied Relative Risk Aversion	Upper Bound	Implied Relative Risk Aversion	Upper Bound
0.08	6.00	2.57	8.72	2.66
0.10	6.17	2.84	8.93	2.94
0.15	6.64	3.54	9.48	3.70
0.20	7.03	4.15	9.94	4.36
0.25	7.37	4.70	10.34	4.97
0.30	7.67	5.21	10.70	5.53
0.35	7.95	5.68	11.02	6.05
0.40	8.19	6.13	11.31	6.56
0.45	8.43	6.56	11.58	7.04
0.50	8.65	6.97	11.83	7.50
0.55	8.84	7.36	12.05	7.95
0.60	9.02	7.75	12.26	8.38
0.65	9.20	8.12	12.46	8.81
0.70	9.36	8.48	12.65	9.22
0.75	9.52	8.83	12.83	9.63
0.80	9.68	9.17	13.01	10.02
0.85	9.80	9.50	13.15	10.41
0.90	9.95	9.83	13.31	10.79
0.95	10.10	10.16	13.47	11.17
1.00	10.20	10.47	13.60	11.53

Table 4.4

This table shows the empirical Jump Diffusion parameters for the S&P 500 Index as measured in the corresponding econometric studies that assume that the underlying process is Jump Diffusion. All the reported parameters are annual. * indicates cases where the reported studies did not estimate the risk-free rate, arbitrarily set at 5%. The differences in jump parameters between EJP and the other studies stems from the fact that EJP captures small jumps with stochastic volatility, which leads to a lower jump intensity and higher mean and volatility of the jumps.

Paper	Dates	Equity Premium	Vol. (σ)	Jump Intensity	Mean Jump	Jump Vol.	Risk Free
Honore (1998)	1928-1988	7.94%	10.04%	62.15	-0.13%	1.9%	5.0%
Andersen et al (2002)	1953-1996	3.22%	9.91%	12.63	0.00%	2.6%	5.1%
Andersen et al (2002)	1980-1996	10.80%	11.38%	14.89	0.00%	3.4%	5.1%
Ramezani and Zeng (2007) ^a	1926-2003	2.56%	13.49%	10.63	0.08%	2.4%	5.0%*
Ramezani and Zeng (2007) ^b	1926-2003	5.08%	12.70%	18.57	0.05%	2.0%	5.0%*
Eraker et al (2003)	1980-1999	7.50%	12.91%	1.51	-2.59%	4.1%	5.0%*

^a Based on raw returns.

^b Based on dividend-adjusted returns.

Table 4.5

This table shows the implied upper bound RRA and corresponding implied equity premium for the studies reported in Table 4.4. Implied upper bound relative risk aversion (column 4) is defined by using the JD upper bound option prices (column 3) together with the equilibrium option prices from the [Bates \[1991\]](#). Implied equity premium is calculated following [Mehra and Prescott \[1985\]](#). The consumption data is annual U.S. data from 1890 to 2004 from [Barro \[2006\]](#), where the growth rate of real consumer expenditure per person has a mean of 0.020 and its standard deviation is 0.035.

Underlying Parameters	Dates	JD Upper Bound Option Prices	Implied Upper Bound Relative Risk Aversion	Implied Equity Premium	Observed Equity Premium
Base Case Parameters		4.70	7.0	0.93%	2.00%
Honore (1998)	1928-1988	5.49	37.5	4.10%	7.94%
Andersen et al (2002)	1953-1996	3.89	26.5	3.52%	3.32%
Andersen et al (2002)	1980-1996	5.82	28.5	3.69%	10.80%
Ramezani and Zeng (2007) ^a	1926-2003	4.13	33.5	3.99%	2.56%
Ramezani and Zeng (2007) ^b	1926-2003	4.47	47.5	3.84%	5.08%
Eraker et al (2003)	1980-1999	4.62	22.5	3.10%	7.50%

^a Based on raw returns.

^b Based on dividend-adjusted returns.

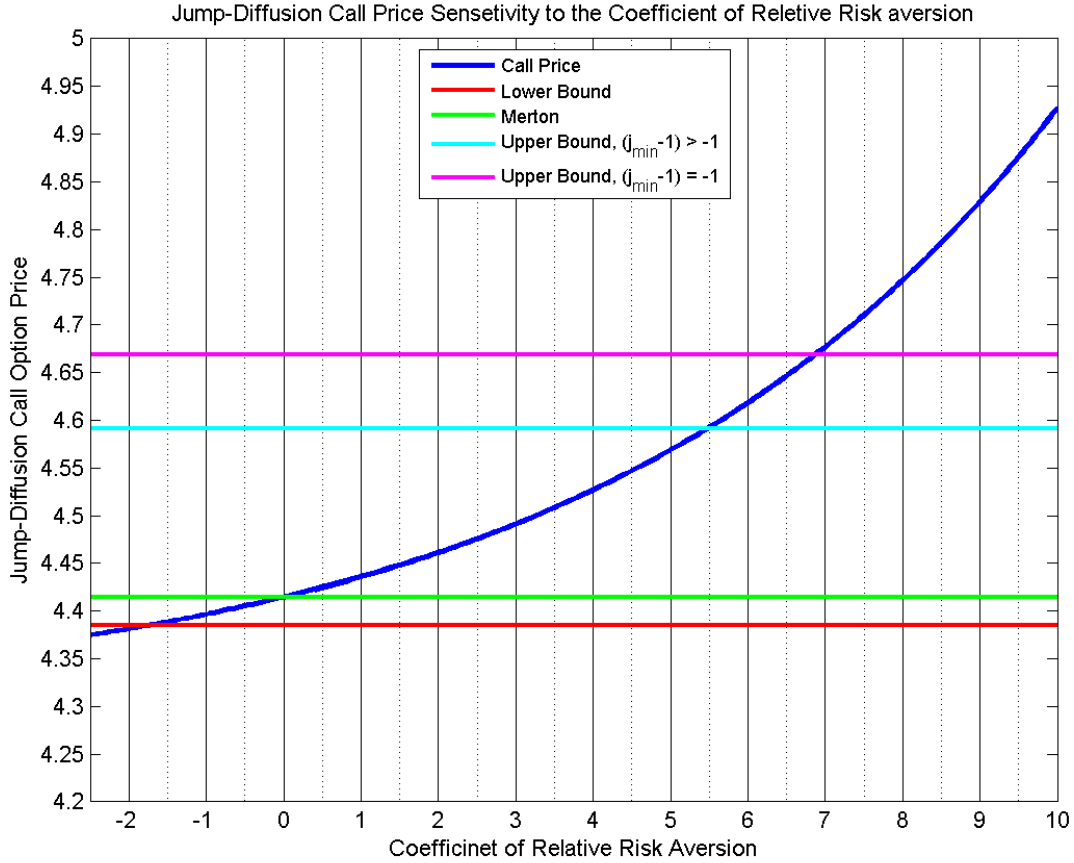


Figure 4.1: JD Call Price Sensitivity to the Coefficient of Relative Risk Aversion

This figure shows the sensitivity of the equilibrium jump-diffusion call option prices to the coefficient of relative risk aversion for a continuum of coefficients up to 10. The parameters are $S = 100$, $X = 100$, $T = 0.25$, $r = 2\%$, $\mu = 4\%$, $\sigma = 20\%$, $\lambda = 0.6$, $\mu_j = -0.05$, $\sigma_j = 7\%$. The price of call option is based on the Bates [1991] jump-diffusion model. In case $j_{min} - 1 > -1$, the upper bound's distribution is truncated to a worst-case jump return of -20% . When $j_{min} - 1 = -1$, the lower limit of the jump amplitude is set to 0. Therefore, the return distribution has an absorbing state where the stock becomes worthless.

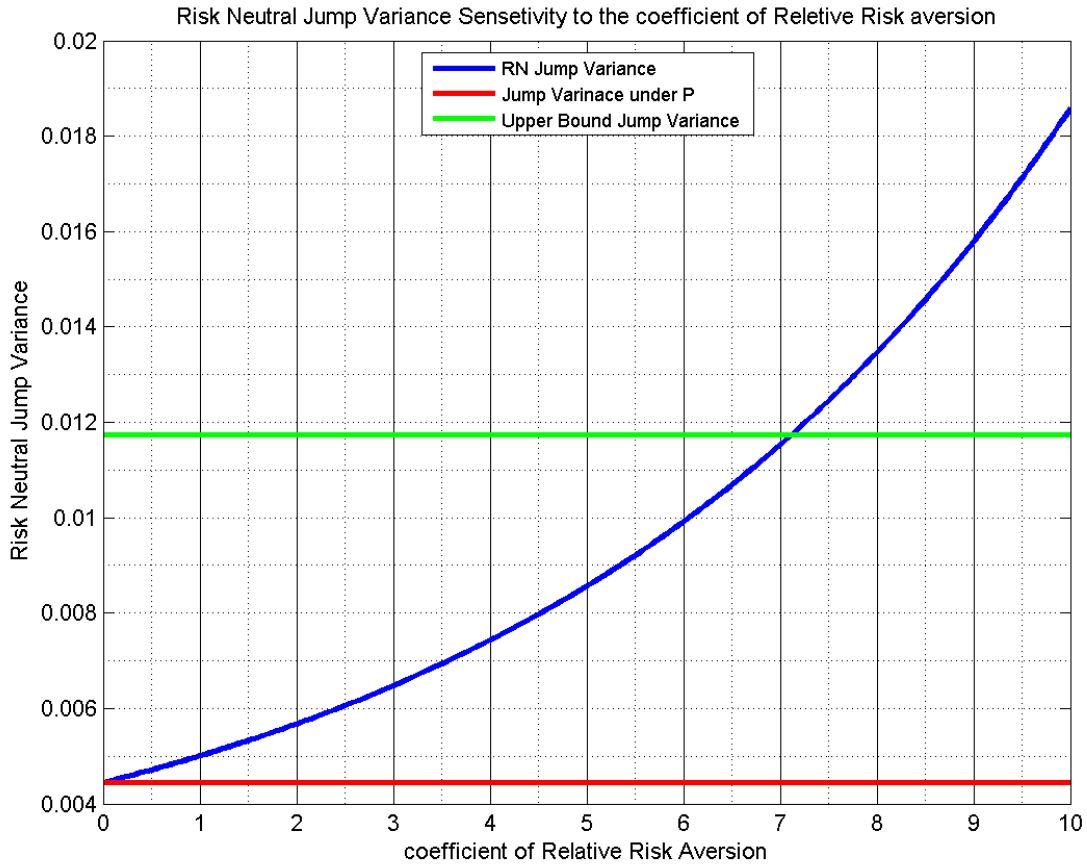


Figure 4.2: Risk_neutral Jump Variance Sensitivity to the Coefficient of Relative Risk Aversion

This figure describes the sensitivity of the variance of the jump component of the log return under Q -distribution to the coefficient of relative risk aversion for a continuum of coefficients up to 10. The parameters are $r = 2\%$, $\mu = 4\%$, $\sigma = 20\%$, $\lambda = 0.6$, $\mu_j = -0.05$, $\sigma_j = 7\%$, following the base case parameters. The upper bound jump variance is defined based on the full support of the jump distribution.

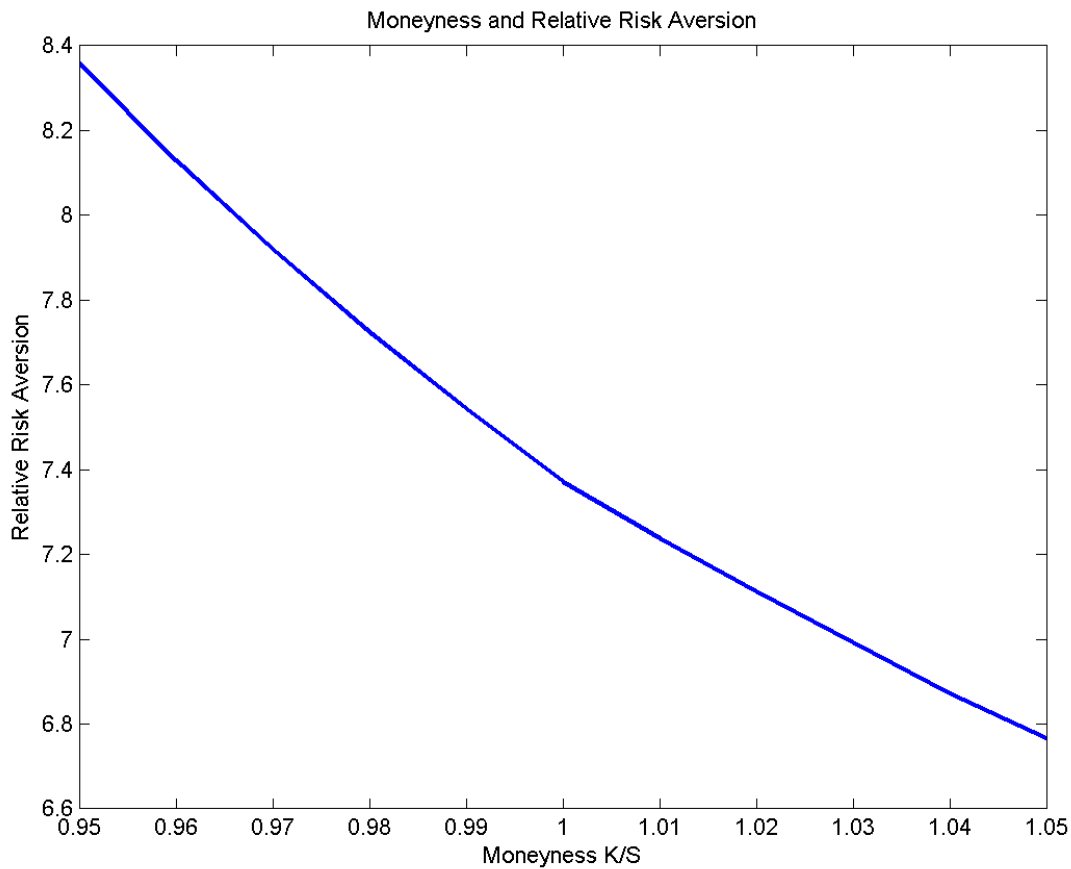


Figure 4.3: Moneyness and Relative Risk Aversion

This figure describes the sensitivity of the SD implied upper bound on the relative risk aversion to the moneyness. SD upper bound on call option prices. SD bound is defined based on the base case parameters $S = 100$, $T = 0.25$, $r = 2\%$, $\mu = 4\%$, $\sigma = 20\%$, $\lambda = 0.6$, $\mu_j = -0.05$, $\sigma_j = 7\%$ on the entire support of the jump distribution.

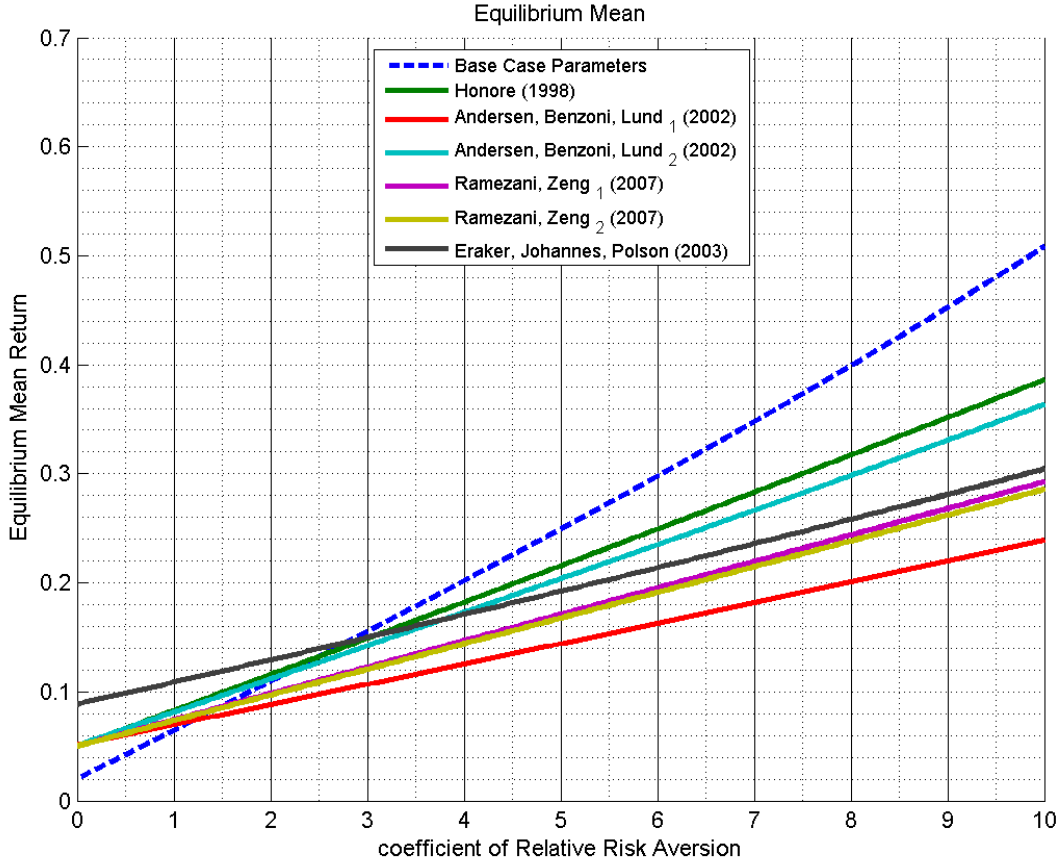


Figure 4.4: Equilibrium Mean Return and Relative Risk Aversion

This figure describes the sensitivity of the equilibrium mean of the jump-diffusion return process to the coefficient of relative risk aversion for a continuum of coefficients up to 10. The base case parameters are $r = 2\%$, $\mu = 4\%$, $\sigma = 20\%$, $\lambda = 0.6$, $\mu_j = -0.05$, $\sigma_j = 7\%$. This relation is based on the well-known equilibrium risk premium where $\mu_t - r = \sigma +_j$ and $\sigma = \gamma\sigma^2$ and $_j = \lambda k - \lambda^Q k^Q$. We draw the equilibrium mean return based on the parameters estimated from underlying S&P 500 index returns in [Honore \[1998\]](#), [Andersen et al. \[2002\]](#), [Ramezani and Zeng \[2007\]](#), and [Eraker et al. \[2003\]](#). More details regarding the underlying parameters are given in Table (4.4).